Evaluation of IGRF 11 SV Candidate Models (W. Kuang and Z. Wei)

There are 8 SV candidate models (labeled alphabetically from A to H) evaluated by our group. The detailed descriptions of each model are provided in the dedicated website.

In our evaluation process, we focus on the following properties (1) the mean $M_n$ and the standard deviation $\sigma_n$ of the energy $R_n$ for each degree $n$, and the distribution of the individual models referenced to $M_n$ and $\sigma_n$; (2) the mean and the standard deviation of each SV coefficient ($g_n^m, h_n^m$) of degree $n$ and order $m$.

For the first part of evaluation, the energy $R_n$ for each degree $n$ is defined as

$$R_n = (n+1)\sum_{m=0}^{n} (g_n^m)^2 + (h_n^m)^2$$  \hspace{1cm} (1)

The mean $M_n$ and the standard deviation $\sigma_n$ for all 8 candidate models are defined as

$$M_n = \frac{1}{8} \sum_{i=1}^{8} R_n^{(i)},$$

$$\sigma_n = \left[ \frac{1}{7} \sum_{i=1}^{8} \left( R_n^{(i)} - M_n \right) \right]^{1/2}.  \hspace{1cm} (2)$$

The results are plotted in Figure 1. From the figure we observe that there are two different sets, (A, B, F, G) and (C, D, E, H): one is mostly above the mean and the other is mostly below the mean. However, we also examine the distribution of the scaled energy spectrum $R_n/M_n$, as shown in Figure 2. In this figure, the relative differences

![Figure 1. The energy spectrum $R_n$ for all SV candidate models.](image-url)
among the models increase with the degree $n$. In particular, the largest relative differences occur for the degrees $n = 3, 6, 7$.

The results on the individual coefficients are different, as shown in Figure 3, where the distributions of the coefficients ($g_n^m$, $h_n^m$) scaled by their means are displayed. From this plot, except the candidate model E (light blue) and G (dark green), the coefficients from the other candidate models are very close to each other.

Our results suggest that to reduce the relative differences among the individual coefficients, and among the energy distribution, candidate models E and G may be weighted differently from the rest of the candidate models.

Figure 2. Similar to Figure 1, but it is the scaled energy spectrum $R_n/M_n$ for all SV candidate models. The scaled results are reset in the plot such that 0 implies the mean value. The two dotted lines are the scaled standard deviation $\pm \sigma_n/M_n$.

Figure 3. Distributions of the spectral coefficients $g_n^m$ (top) and $h_n^m$ (bottom) scaled by their means. Again, the dashed lines are the scaled standard deviation for each degree $n$ and order $m$. 