Sheath Interference Effects in the Bare-tether Array of an Electric Solar Sail

Juan R. Sanmartin, Antonio Sanchez-Torres,

Shaker B. Khan,

Universidad Politecnica de Madrid

Spacecraft Charging Technology Conference, Sept 20-24, 2010, Albuquerque, NM

1 E-sail concept

Electric Solar Sail (e-sail) is new technology (P. Janhunen, JPP 2004) using the dynamic pressure of the solar wind for propulsion. Standard solar sails use a physical membrane whereas e-sails would use a bare-tether array to set up a virtual sail.

At 1AU the photon pressure is 3 orders of magnitude larger than the dynamic pressure in the solar wind,

$$P_{dyn} \approx m_i n_i v_{sw}^2 \sim 2\text{nN}/\text{m}^2, \qquad P_{phot} \approx 4.563 \cdot \left(\frac{1\text{AU}}{\text{r}}\right)^2 \frac{\mu\text{N}}{\text{m}^2}$$

with a solar constant

$$S \equiv P_{phot} \times c \approx 1.27\ \text{kW}/\text{m}^2 \times (1\text{AU}/r)^2$$

However, an e-sail may have a (virtual) effective area per unit mass much larger than a standard sail membrane; the e-sail thrust might be comparatively much larger.

2 Ambient Solar Wind conditions

Typical values for the ambient plasma at 1AU in the ecliptic plane are

$T_e \sim 12$ eV, $B \sim 10$ nT, $n \sim 7$ cm^{-3}, $v_{sw} \approx 400$ Km/s, H^+ ions

Density models (Leblanc et al. 1996) become simply $n \propto 1/r^2$ beyond 1AU

$$n \approx 7.2\left(\frac{1\text{AU}}{r}\right)^2 + 1.95\cdot 10^{-3}\left(\frac{1\text{AU}}{r}\right)^4 + 8.1\cdot 10^{-7}\left(\frac{1\text{AU}}{r}\right)^6 \text{cm}^{-3} \approx 7.2\left(\frac{1\text{AU}}{r}\right)^2$$

Different rough T_e models may be used beyond 1AU

$$T_e \approx 3.8\times\left[1+1.5\left(\frac{1\text{AU}}{r}\right)^{4/3}\right]\text{eV}, \qquad T_e \approx 12\times\left(\frac{1AU}{r}\right)^{1/3} eV$$

In a simple isothermal outflow model, the solar wind velocity increases slowly with distance from the Sun, $v_{sw}^2 \propto \ln r$

Two lengths are basic parameters in considering tether use in LEO and at 1AU

$v_{rel} / \Omega_i \quad (\Omega_i \equiv eB/m_i)$ and λ_D

* As regards v_{rel} / Ω_i

In LEO, $v_{rel} \approx$ 8km/s, $B \sim$ 0.3 Gauss, O^+ ions $\Rightarrow$ $v_{rel} / \Omega_i \sim 10^2$ m

At 1AU, $v_{rel} \approx$ 400km/s, $B \sim 10^{-4}$ Gauss, H^+ ions $\Rightarrow$ $v_{rel} / \Omega_i \sim 10^6$ m

$\Omega_i L / v_{rel}$ will be typically small at $1\mathrm{AU}$ and large in LEO

* As regards λ_D

In LEO, $T_e = 0.15\,\mathrm{eV}$, $n = 10^5\text{-}10^6\ \mathrm{cm}^{-3} \Rightarrow \lambda_D \sim$ 5 mm (around practical R values)

At 1AU, $T_e = 12\,\mathrm{eV}$, $n < 10\ \mathrm{cm}^{-3} \Rightarrow \lambda_D \sim$ 10m >> R

Coulomb-to-Lorentz force ratio

$F_C \sim 2\, r_{sh}\, L \times n m_i v_{rel}^2$ (r_{sh} = sheath radius)

$$F_L \sim I_{av}\, LB, \qquad I_{av} = \frac{1}{2}\, 2RL \times en \sqrt{\frac{2e\Delta V}{m_e}}$$

The very small magnetic field at 1AU makes bias ΔV uniform on a tether

(negligible induced emf)

Consider positive bias ($\Delta V > 0$), repelling ions / attracting electrons

The Coulomb-to-Lorentz force ratio reads

$$\frac{F_C}{F_L} \sim \frac{v_{rel}}{\Omega_i L} \frac{\lambda_D}{R} \frac{r_{sh}}{\lambda_D} \sqrt{\frac{m_e}{m_i}} \sqrt{\frac{m_i v_{rel}^2 / 2}{e\Delta V}}$$

The Coulomb force, <u>determined by a front-area</u>, is dominant at 1AU, negligible in LEO

In principle, $e\Delta V$ should be of the order of $\frac{1}{2} m_i v_{rel}^2$

With $I_{\max} = 2 I_{av} \propto n\sqrt{\Delta V}$

the power required for a given bias is $\dot{W} = I_{\max}\Delta V \propto n\Delta V\sqrt{\Delta V}$

Both provided power $\dot{W} = A(\text{solar panel}) \times S$, $\dot{W} \propto S$

and density n itself,

vary with distance to the Sun as $1/r^2$ making

$$\Delta V \propto \left(\frac{\dot{W}}{n}\right)^{2/3}$$ Independent of r

The ratio $\frac{e\Delta V}{m_i v_{rel}^2/2}$ and the propulsive efficiency might keep constant

with increasing r if the SC velocity increases along with v_{sw}

3 Single tether e-sail

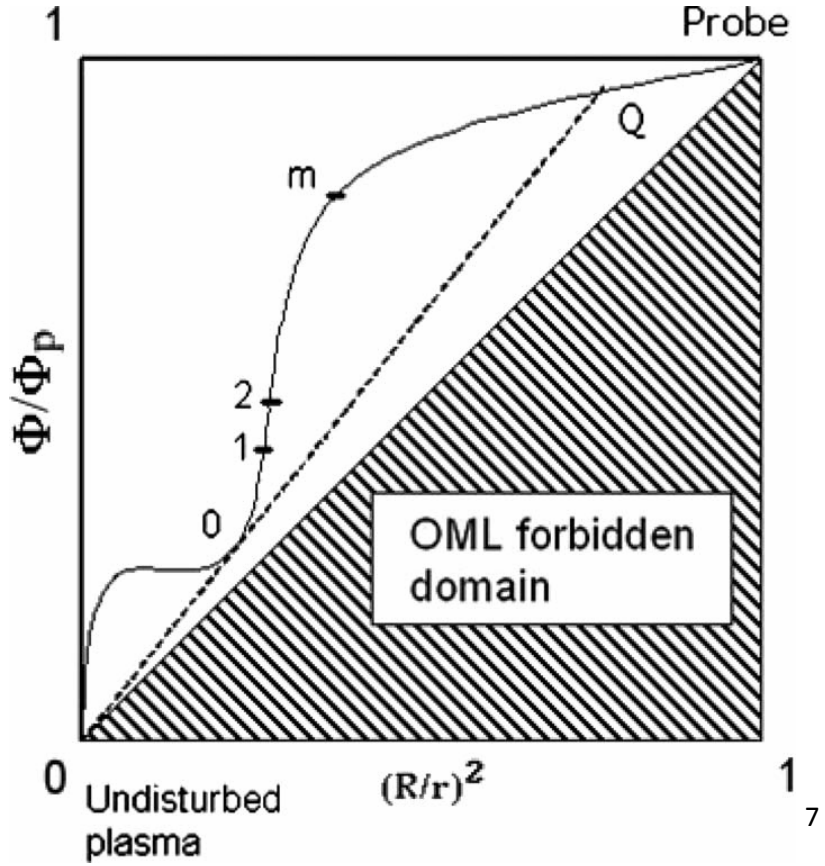
Potential profile of a cylindrical probe at high bias, in unmagnetized plasma in thermal equilibrium ($T_i = T_e$): Potential vs $\frac{1}{r^2}$ behavior

The sheath starts at $r_{sh} \sim r_1 \sim r_2$

A relation to be used later reads

$$\frac{R^2}{r_{sh}^2} \frac{e\Phi_p}{kT} \approx \sigma_1 \left(\frac{R}{\lambda_D} \right)$$

where σ_1 is O(1) for $R \sim \lambda_D$ and small (large) for R / λ_D small (large)



Bare-tether sheath law

At the very high bias of interest, $e\phi_p >> kT$, we have $r_{sh} >> \lambda_D$

Using the approximation

$$\frac{r_2}{r_1} = 1 - \frac{3.45}{(\lambda\mu)^{1/5}}\left(\frac{\lambda_D}{r_{sh}}\right)^{4/5} \approx 1$$

the sheath law yielding r_{sh} / λ_D versus $e\phi_p / kT$ and R / λ_D reads

$$0.974\left(\kappa\sqrt{\sigma_1}\right)^{2/3}\left(\frac{r_{sh}}{\lambda_D}\right)^{4/3}\ln\left(0.704\times\frac{r_{sh}/\lambda_D}{R/\lambda_D}\right) = \frac{e\Phi_p}{kT}$$

(λ, μ and κ are functions of R / λ_D like σ_1 itself)

Sheath law valid for any R/λ_D, using corresponding values of κ and σ_1

$0.974\,(\kappa\sqrt{\sigma_1})^{2/3} \approx 1.53$ (and $\lambda\mu = 1.168$) for the $R << \lambda_D$ e- sail case

At $R = R_{max}$ for OML validity ($\sim \lambda_D$), <u>of interest</u> for e-sail tether-interference

$$0.974\,(\kappa\sqrt{\sigma_1})^{2/3} \approx 1.37 \qquad \text{(and } \lambda\mu = 5.0, \quad \sigma_1 \approx 0.24\text{)}$$

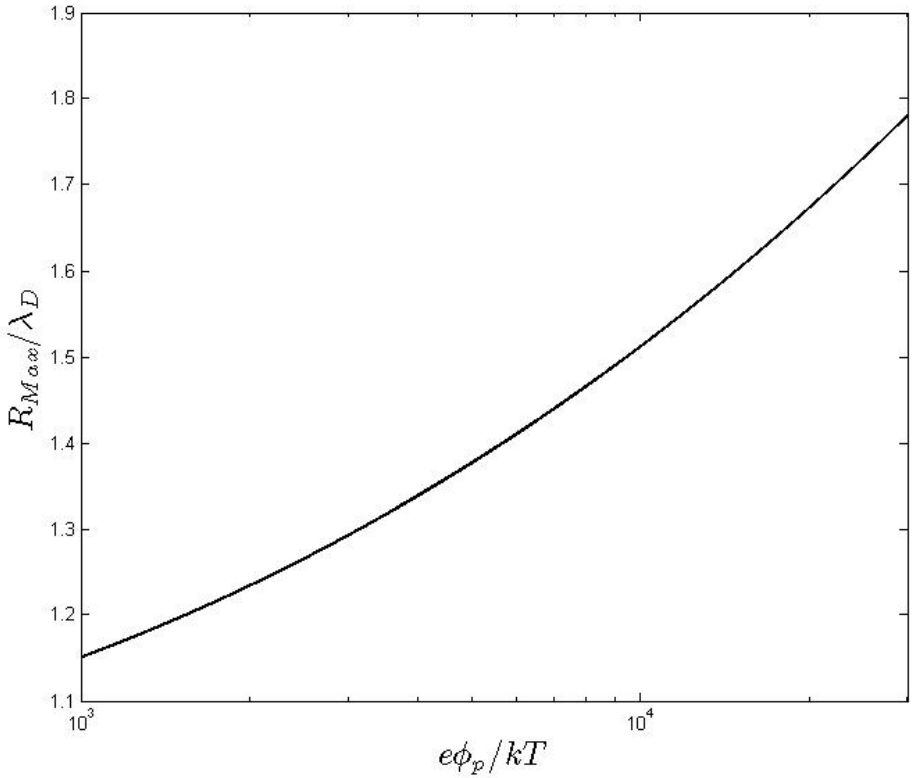
Using $0.24\left(\dfrac{r_{sh}}{R_{\max}}\right)^2 \approx \dfrac{e\Phi_p}{kT}$

in the R_{max}-sheath law

yields r_{sh}/λ_D vs just $e\phi_p/kT$ (>> 1)

$$\sqrt{0.24} \times \frac{r_{sh}}{\lambda_D} = \left(\frac{0.564 \times e\Phi_p / kT}{\ln(2.06 \times e\Phi_p / kT)}\right)^{3/4}$$

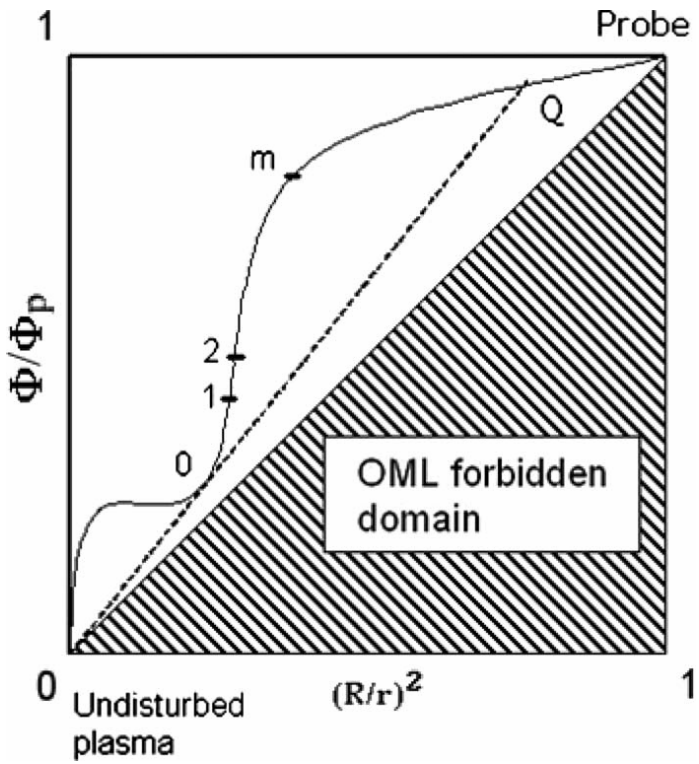
and R_{max}/λ_D vs $e\phi_p/kT$ as shown



Profile point m corresponds to a maximum of ϕr^2 and a minimum of density in the sheath. It marks a fast drop slightly beyond r_m

$$r_m = r_{sh} e^{-0.63} \approx 0.53 r_{sh},$$

$$e\Phi_m = \frac{0.41 \times e\Phi_p}{\ln\left(0.704 \times \frac{r_{sh}}{R}\right)} >> e\Phi_{sh} \sim kT, \qquad \Phi_p \equiv \Delta V$$



In the Coulomb force $F_C \propto r_m n v_{rel}^2$

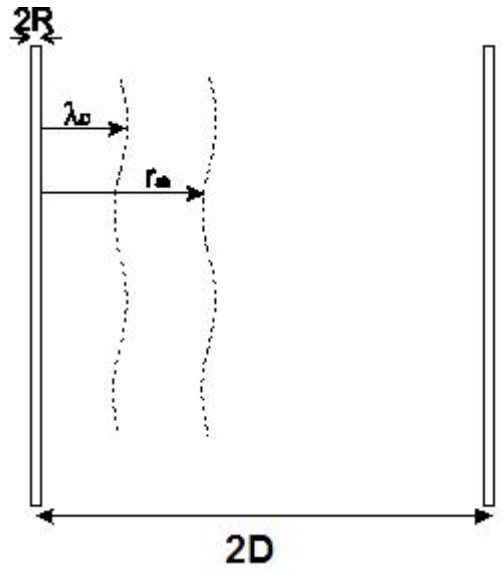
$$r_m \propto r_{sh} \propto \lambda_D \times f\,(e\Delta V / kT,\ R/\lambda_D)$$

$$\Rightarrow \quad F_C \propto v_{rel}^2 \times \sqrt{nT} \times f\,(e\Delta V / kT,\ R/\lambda_D),$$

decreasing when moving away from the Sun

Note that f increases with R: Sheath interference between 2 close wires might result in an equivalent ‘single’ tether with net gain in sheath size.

4 Tether array / Multiline-tether



Interference in current-collection / sheath-structure may occur within a tether array and within a 'multiline' tether' (not really 'single' but made of interconnected wires for survival)

At high bias, a tether with $R \sim R_{max} \sim \lambda_D$ is surrounded by a region free of space-charge effects.

2 tethers at distance D <u>not large</u> compared with λ_D behave as single tether with 'equivalent' radius $R_{eq} \sim D$ (determined by Laplace's equation) at distances large compared with R_{eq}

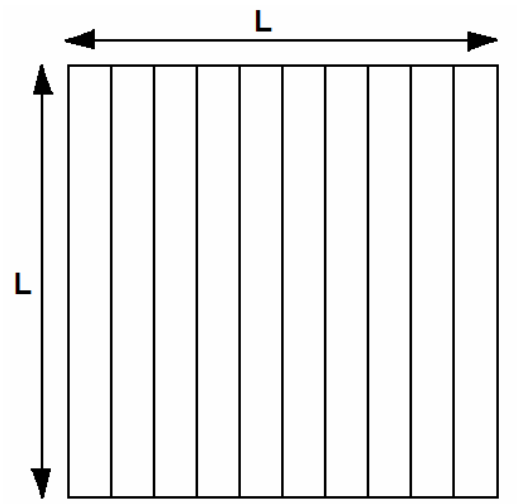
Interference involves 2 respects: Is the 'equivalent' tether simpler? Does it show a net gain in sheath size? Values of D of interest are comparable to R in multiline tethers, larger than λ_D for arrays.

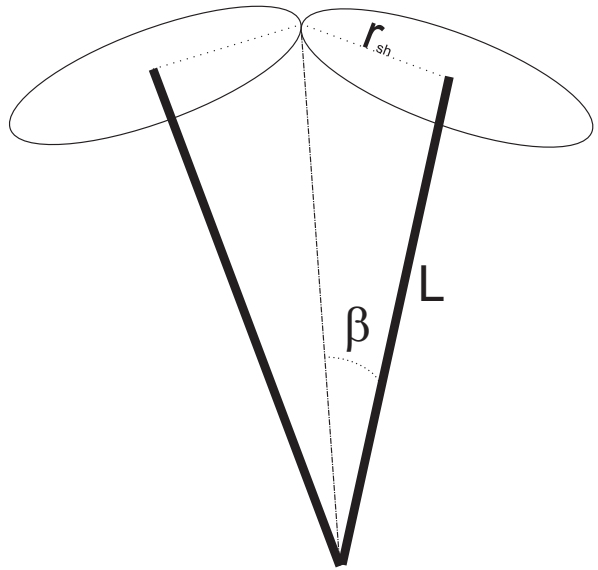
In a Starfish-like array, N tethers meet at the spacecraft / power system

At the opposite ends, sheaths of adjacent tethers are assumed to just not overlap

$N = \frac{\pi}{\beta} \approx \pi \frac{L}{r_m} >> \mathbf{1}, \quad \beta = \tan^{-1}\frac{r_m}{L}$ (Tethers are near parallel)

Sheaths just interfere at points closer to the SC





$\rho \equiv$ mass density

A parallel array has half the mass per unit area ($\rho\pi R^2 / 2r_m$ as against $\rho\pi R^2 / r_m$)

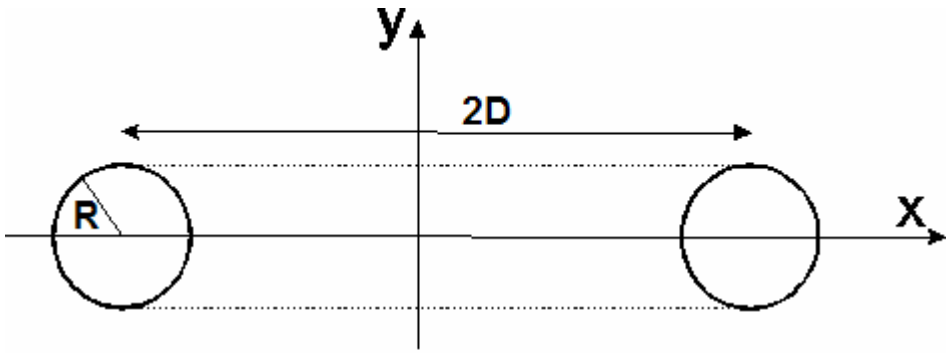
It may allow increasing the sheath front area by playing with distance between tethers

Consider the potential at the origin for the Laplace solution or a two-circle equally charged, conductive cylinder,

with potential vanishing at a centered circle of large radius r_∞

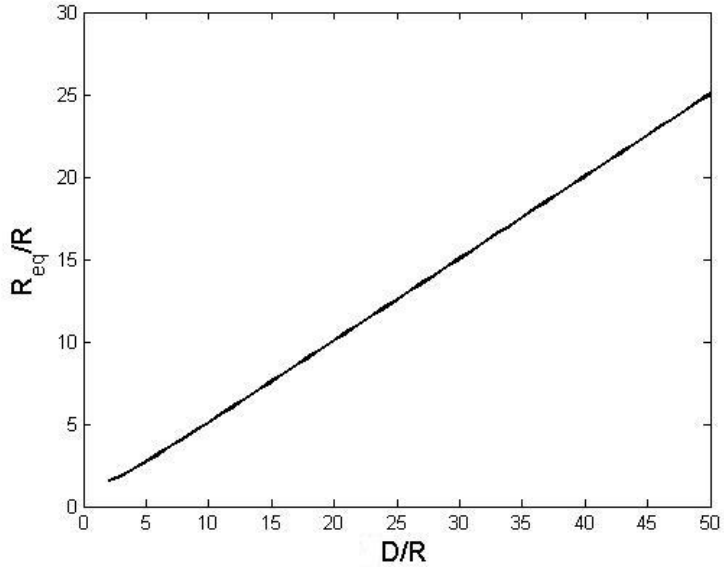
$$\Phi(0) = \frac{2q}{2\pi\varepsilon_0}\left[\ln\frac{r_\infty}{D} + \ln\left(\frac{D-\lambda_1 R}{D}\right) + \sum_1^\infty \ln\frac{(D-\lambda_{2k-1}R)(D-\lambda_{2k+1}R)}{(D-\lambda_{2k}R)^2}\right]$$

$$\lambda_1 \equiv \frac{R}{2D}, \qquad \frac{1}{\lambda_k} = \frac{1}{\lambda_1} - \lambda_{k-1} \quad (k = 2,3,\ldots)$$



The capacity per unit length $C \equiv \frac{2q}{\Phi(0)} = \frac{2\pi\varepsilon_0}{\ln(r_\infty / R_{eq})}$ yields the radius R_{eq} of a single cylinder behaving as the two-circle cylinder for $r >> R_{eq}$

$$R_{eq} = \frac{D}{1-2\lambda_1^2} \times \prod_1^\infty \frac{(1-2\lambda_1\lambda_{2k})^2}{(1-2\lambda_1\lambda_{2k-1})(1-2\lambda_1\lambda_{2k+1})}$$



For $D = R$, $R_{eq} = \pi D / 2$ exactly

For $D \geq 2R$, $R_{eq} \approx \frac{D}{1-(R^2/2D^2)}$

For $D = 2R$ and $3R$ the equivalent radius is 1.14 D and $1.06\,D$ respectively

It rapidly approaches $R_{eq} \approx D$ with increasing D / R

Two tethers well separated contribute $L \times 4r_{sh}$ front area, where r_{sh} follows

from the $R \ll \lambda_D$ sheath law, $$1.53 \times \ln\left(0.704 \times \frac{r_{sh}/\lambda_D}{R/\lambda_D}\right)\left(\frac{r_{sh}}{\lambda_D}\right)^{4/3} = \frac{e\phi_p}{kT}$$

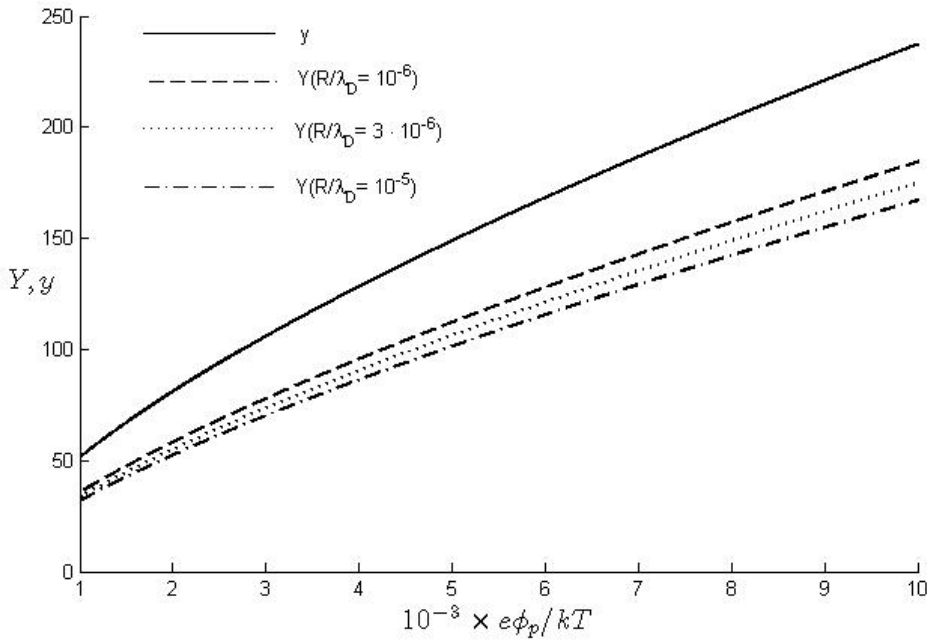
If at distance $R_{max} \sim \lambda_D$, however, they contribute $L \times 2r_{sh}(R_{max})$ where

$$\sqrt{0.24} \times \frac{r_{sh}(R_{\max})}{\lambda_D} = \left(\frac{0.564 \times e\Phi_p/kT}{\ln(2.06 \times e\Phi_p/kT)}\right)^{3/4}$$

The figure shows

$$y \equiv r_{sh}(R_{max})/\lambda_D$$

$Y \equiv 2r_{sh}(R)/\lambda_D$ for 3 R/λ_D values



At given bias ratio $e\Phi_p / kT$ and Debye length λ_D , a characteristic increase in ‘tether’ radius by a factor of order of 10^6, when setting a distance $\sim \lambda_D$ between a pair of wires, yields a sheath radius that is more than twice the sheath radius for each single wire

Detailed numerical caculations should allow setting distance within a pair beyond one Debye length

Pairing tethers in a parallel array, with distance about λ_D within each pair, and distance between pairs about their resulting r_{sh}, will produce a net front-area gain for the array. No such gain seems possible in a starfish array

Interference between wires in a multiline tether, where distances would be much less than λ_D, would mainly affect current-collection, and indirectly the power system

5 Conclusions

At 1AU, the Coulomb force, which depends on sheath size r_{sh}, is much larger than the Lorentz force because of both weak B and large λ_D

Though depending logarithmically on tether radius, r_{sh} increases sensibly in going from actual values $R \sim 30\mu m$ to about $\lambda_D \sim 10$ m

At the high bias of interest, space charge has no effects in a region reaching beyond λ_D around a tether, where Laplace's equation holds

Sheath interference between two $R \sim 30\mu m$ wires at distance $\sim \lambda_D$ will result in a (Laplace equivalent) 'single' tether with net gain in sheath size.

Pairing tethers in a parallel array, with distance about λ_D within each pair, and distance about their resulting r_{sh} between pairs, will produce a net front-area gain for the array

References

[1] Janhunen, P. and Sandroos, A., “Simulation study of solar wind push on a charged wire: basis of solar wind electric sail propulsion”, Ann. Geophys., 25, 755-767, 2007.

[2] Janhunen, P., “On the feasibility of a negative polarity electric sail”, Ann. Geophys., 27, 1439-1447, 2009.

[3] Sanmartín, J. R., Choinière, E., Gilchrist, B. E., Ferry, J. B., and Martínez-Sánchez, M., “Bare-tether sheath and current: comparison of asymptotic theory and kinetic simulations in stationary plasma”, IEEE Trans. Plasma Science., 36, 2851-2858, 2008.

[4] Sanmartin, J. R. and Estes, R. D., “Interference of parallel cylindrical Langmuir probes”, Phys. of Plasmas, Vol. 8, 9, 2001.

[5] Forward, R. L. and Hoyt, R. P., “Failsafe Multiline Hoytether Lifetime”, AIAA, 1995.