As of 10 June 2016

This GOES-R Level 2+ space weather Algorithm Theoretical Basis Document (ATBD) is preliminary and subject to change as the GOES-R Program prepares for a launch in late 2016.

The NOAA National Centers for Environmental Information (NCEI) is currently developing a demonstration version of the Satellite Product Analysis and Distribution Enterprise System (SPADES) which will host the L2+ algorithms. The operational SPADES will be instantiated by the National Weather Service in FY2017 to serve the needs of the Space Weather Prediction Center (SWPC).

Updated versions of the L2+ ATBDs will periodically be made available and posted to the SPADES website.

Bill Denig
NCEI

http://www.ngdc.noaa.gov/stp/space-weather/online-publications/stp_sii/spades/

NOAA NESDIS CENTER for SATELLITE APPLICATIONS and RESEARCH

MAGNETOMETER CONVERT DATA TO ALTERNATE GEOPHYSICAL COORDINATE SYSTEMS ALGORITHM THEORETICAL BASIS DOCUMENT
Version 1.3

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TITLE: MAGNETOMETER CONVERT DATA TO ALTERNATE GEOPHYSICAL COORDINATE SYSTEMS ALGORITHM THEORETICAL BASIS DOCUMENT
VERSION
1.3

AUTHORS:
Dr. Paul Lotoaniu
Dr. Howard Singer

APPROVAL SIGNATURES:

_____<Actual Signature Date>
Mary Shouldis
Technical Approver

_____<Actual Signature Date>
Howard Singer
Agency

MAG CONVERT DATA TO ALTERNATE GEOPHYSICAL COORDINATE SYSTEMS ALGORITHM THEORETICAL BASIS DOCUMENT VERSION HISTORY SUMMARY

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1.0	Created by Dr. Paul Lotoaniu CIRES/SWPC	New Document	Feb 12, 2008
1.2	Revised by Paul Loto'aniu	All	Mar, 2009
1.3	Revised by Paul Loto'aniu	All	Dec, 2009

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LIST OF ACRONYMS

ATBD	Algorithm Theoretical Basis Document
CIRES	Cooperative Institute for Research in Environmental Sciences
MAG	Magnetomer
STAR	Center for Satellite Applications and Research
SWPC	Space Weather Prediction Center

ABSTRACT

The algorithm converts Level 1B GEI coordinate MAG data including inclination and total magnetic field into local magnetic (VDH), geocentric solar ecliptic (GSE) and geocentric solar magnetospheric (GSM) coordinate systems. Each coordinate system is defined in 3-dimensions with a schematic for visualization. Mathematical and pseudo code descriptions are given of each coordinate transformation algorithm. Described are assumptions made regarding input science data, instrumentation and input spacecraft attitude data and ephemerides. A description is given of geomagnetic dipole calculation based on IGRF2005 and Sun location using J2000 epoch. Example test runs using proxy data are shown and compared to results from currently implemented algorithms and literature, with comparisons showing excellent agreement. Pre- and post-launch cal/val plans are presented along with current algorithm limitations and possible future improvements.

1.0 INTRODUCTION

1.1 Purpose of This Document

The purpose of this document is to describe the product algorithm and its theoretical basis. This document can be used as a reference for implementing the algorithm into software and as a guide to geocentric coordinate systems important in space weather. The coordinate transformation product converts Level 1B magnetometer data into different geophysical coordinate systems. Multiple coordinate systems are needed because different physical phenomena in space weather can best be understood or described in different systems. These coordinate systems are local magnetic (VDH), geocentric solar ecliptic (GSE), geocentric solar magnetospheric (GSM) and geocentric equatorial inertial (GEI). The algorithms are partially based on the work of Russell (1971) and Hapgood (1992; 1995; 1997). Each coordinate system is defined in 3-dimensions with a schematic for visualization. Mathematical and pseudo code descriptions are given of each coordinate transformation algorithm. The VDH and GSM coordinates require knowledge for the geomagnetic dipole and this is calculated using the International Geomagnetic Reference Field (IGRF) 2005 model. Calculation of the Sun location, which is required for conversion to GEI, GSE and GSM coordinates are based on J2000 epoch. The assumptions made regarding input data, instrumentation and input spacecraft attitude are specified.

1.2 Who Should Use This Document

The STAR AIT group shall use this document to integrate the algorithm into their collaborative framework environment. It shall also be used by the prime development and implementation contractor to design, develop, test, validate and implement the algorithm into the final operational processing system. In addition, the GOES-R AWG space weather application team shall use this ATBD to verify their operational requirements are being met by the proposed algorithm. They should also use it to understand the strengths and weaknesses of the algorithm as well as its accuracy and applicability.

1.3 Inside Each Section

Section 2.0, OBSERVING SYSTEM OVERVIEW, describes the product objectives and requirements, and assumptions made regarding magnetometer characteristics.

Section 3.0 ALGORITHM DESCRIPTION, describes the theoretical basis of the algorithm, assumptions made for sensory data, input data and ancillary data, algorithm input/output, error estimates and programming, procedural and computational considerations.

Section 4.0 TEST DATA SETS AND OUTPUTS, describes the test data sets used to characterize the performance of the algorithm and the data product quality. Also are described are the results from the algorithm processing on simulated input data.

Section 5.0 PRACTICAL CONSIDERATIONS, discusses issues involving numerical computation, programming and procedures, quality assessment and diagnostics and exception handling.

Section 6.0 ASSUMPTIONS AND LIMITATIONS, discusses algorithm performance with regard to accuracy and the limitations of the algorithm and possible improvements.

Section 7.0 REFERENCES, provides all references mentioned in the ATBD.

1.4 Related Documents

(1) GOES-R MAGNETOMETER PORD (417-R-MAGPORD).

(2) GOES-R Mission Requirements Document 2B Prime (MRD-2B').

(3) C.T. Russell, (1971) "Geophysical Coordinate Transformations", *Cosmic. Electrodyn.* **2**, 184-196.

(4) M.A. Hapgood, (1992) "Space Physics Coordinate Transformations: A User Guide", *Planet. Space Sci.* **40**, 711-717.

(5) M.A. Hapgood, (1995) "Space physics coordinate transformations: the role of precession", *Annales Geophysicae*, **13**, 713-716.

(6) M.A. Hapgood, (1997) "Corrigendum to Space Physics Coordinate Transformations: A User Guide", *Planet. Space Sci.* **45**, 1047.

1.5 Revision History

Revision Number	Date	Author	Revision Description	Reason for Revision
1.0	Feb 12, 2008	Paul Loto'aniu	Create document	Create document
1.2	Mar, 2009	Paul Loto'aniu	Revised all sections	Revised all sections
1.3	Dec, 2009	Paul Loto'aniu	Revised all sections	Revised all sections

2.0 OBSERVING SYSTEM OVERVIEW

2.1 Product Generated

The MAG Convert Data to Alternate Geophysical Coordinate Systems algorithm will convert Level 1B MAG data, including inclination and total magnetic field into local magnetic (VDH), geocentric solar ecliptic (GSE) and geocentric solar magnetospheric and (GSM). The data input is assumed to be in geocentric equatorial inertial (GEI) coordinate systems.

The need for use of more than one coordinate system arises because different physical phenomena in near Earth space can best be understood or described in different coordinate systems. All the alternate coordinate systems are used extensively in the space physics and space weather community. For example, the magnetic coordinates VDH and GSM are convenient for analyzing distortions of the magnetic field configuration such as field line stretching and dayside compression. These distortions are often signatures of strong geomagnetic activity. In addition, the GSM system is regularly used in model validation. The GSE and GSM are very useful for studying solar wind-magnetospheric interaction, while the GEI coordinates provide a fixed star reference.

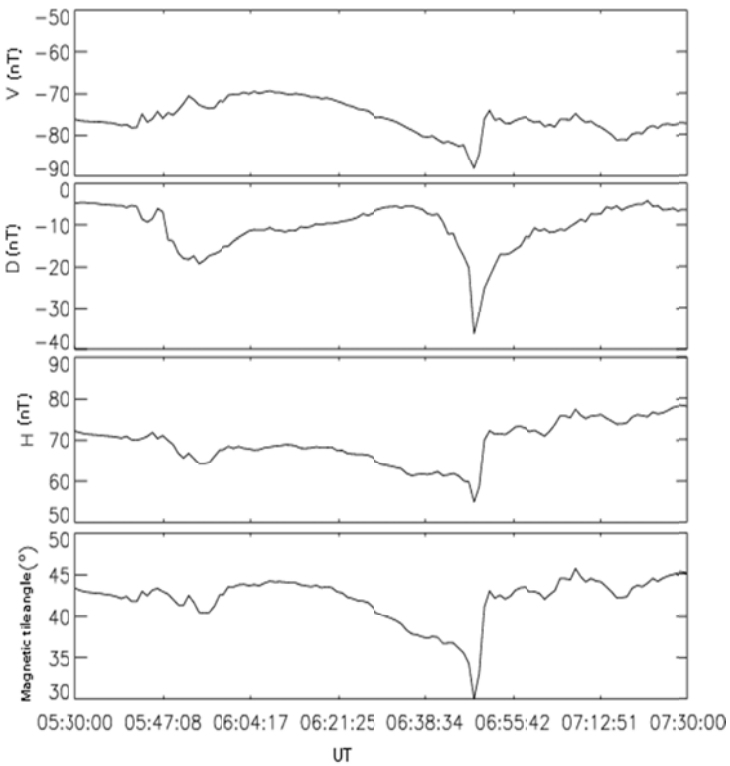

Figure 2.1 Time Series Data in Local Magnetic VDH Coordinates
The magnetic tilt angle or inclination can be calculated using all three VDH components and indicates strong stretching of the magnetic field on the nightside, which maximizes about 06:48 UT than quickly reconfigures back into a more dipole like configuration, known as dipolarization [see, Liou et al., 2001 for analysis of this event].

2.2 Instrument Characteristics

The MAG instrument operational requirements and characteristics are detailed in section 3.4.2.7 of the GOES-R Series Mission Requirements Document (MRD) Version 3.0 dated February 2007 and the MAG instrument Performance and Operational Requirements Document (PORD) draft dated November 2004. The accuracy of the algorithm relies on precise instrument orientation and calibration, data timing and knowledge of spacecraft attitude and location.

Accurate calibration is critical as errors flow through to the coordinate transformed product output. Stray magnetic fields not accounted for in pre-launch ground testing may create offset errors. Attitude changes such as Yaw flips and magnetometer orientation errors will affect the algorithm as they change the definition of axes.

Algorithm development by the SWx AT will assume the instrument meets the performance requirements outlined in the GOES-R MRD and MAG PORD.

3.0 ALGORITHM DESCRIPTION

3.1 Algorithm Overview

The MAG Convert Data to Alternate Geophysical Coordinate Systems algorithms convert Level 1B MAG data in geocentric equatorial inertial (GEI) coordinate systems coordinates and inclination and total magnetic field into local magnetic (VDH), geocentric solar ecliptic (GSE) and geocentric solar magnetospheric (GSM). The algorithms are partially based on the work of Russell (1971) and Hapgood (1992, 1995, 1997).

3.2 Processing Outline

The processing outline of the Convert Data to Alternate Geophysical Coordinate Systems algorithms is summarized in the figure 3.1 below. The current algorithms are implemented as C++ routines with an object oriented emphasis. However, current maturity does not meet ISO 1998 or 2003 standard.

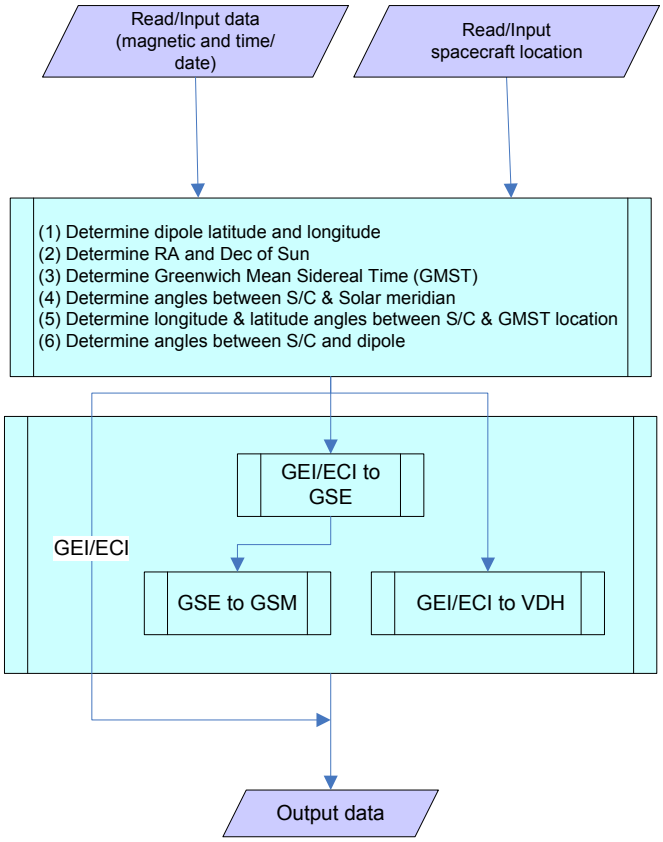

Figure 3.1 Flowchart of the MAG Coordinate Transformation Algorithms

The algorithm processes each line of data assuming an ASCII input data format that contains date, time and three magnetic field values. Coordinate transformation is handled by a transformation object, Tform, which declares and initializes transformation member functions and variables. Tform inherits a dipole (igrf) object which handles geomagnetic dipole location calculations. The dipole location is re-calculated only at the beginning of each day (0 UT).

The initial date and time is used to initiate the date object. Satellite ephemeris information is used to estimate time-dependent rotational angles for the coordinate transformation. Rotational matrices are calculated (see section 3.4) and data undergo coordinate transformation before being outputted.

3.3 Algorithm Input

This section describes the input needed for the coordinate transformation algorithms. The algorithms require instrument sensor data, date and time and spacecraft location. It is assumed that data has been corrected for spacecraft and instrument orientation. Besides the input data, dipole location and Sun location are needed.

3.3.1 Primary Sensor Data

Primary sensor data is defined as information that is derived solely from the MAG observations, date/time tags and spacecraft location. The algorithms assume

- That the sensor data will be three orthogonal components of the fluxgate magnetometer already converted into GEI coordinates.
- That data includes date and time tags.
    - Time is assumed to be at millisecond accuracy.
    - Date is day and month in two digit integer year as for four digit integer.
- That data includes spacecraft location in geographic longitude and latitude and spacecraft altitude.

3.3.2 Ancillary Data

There is no ancillary data. However, the geomagnetic dipole location and Sun location are required for the transformations. For example, the VDH and GSM coordinates require knowledge of the geomagnetic dipole, while calculation of the Sun location is required for conversion to GEI, GSE and GSM. Requirements are

- Geomagnetic dipole location in geographic latitude and longitude.
- Sun location at date/time of observation and Vernal equinox geographic longitude.

3.4 Theoretical Description

3.4.1 Physics of the Problem

In this section we give definitions of all the coordinate systems of importance to the algorithms. The transformations cannot be described without mathematical equations. Hence, we discuss the transformations in section 3.4.2. Output data will be transformed into only four coordinate systems. However, seven systems are described here including the coordinate system of the input data and two intermediate systems required for transformations.

3.4.1.2 GLS (RH-ENP) System

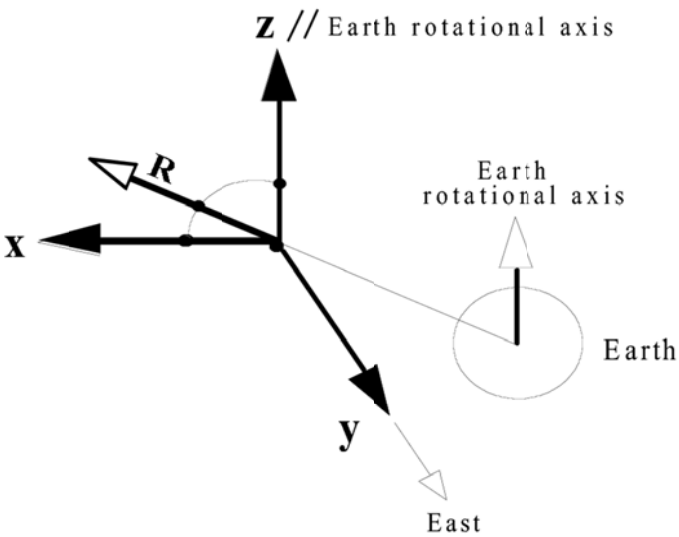

Figure 3.3 Geocentric Local Spacecraft Coordinate System
This system is a local coordinate system, which is dependent of the position of the point of observation from the Earth. This is an intermediate coordinate system required for all transformations. Note: In c++ code, the GLS coordinates is referred to as the RH-ENP or RHENP coordinates.

3.4.1.3 Local Magnetic VDH System or Dipole Meridian (DM) System

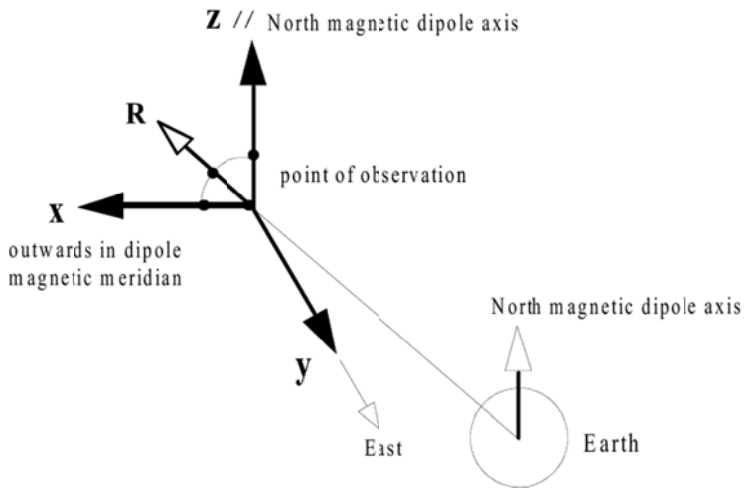

Figure 3.4 Local Magnetic VDH or DM System
Z (H) is parallel to the North magnetic dipole axis. The X (V)-Z (H) plane contains the direction **R** of the point of observation, from the Earth, and is a dipole magnetic meridian plane. The Y(D)-axis is perpendicular to the **R** vector, eastwards. This system is a local coordinate system, which is dependent of the position of the point of observation from the Earth.

3.4.1.4 Geographic (GEO) System

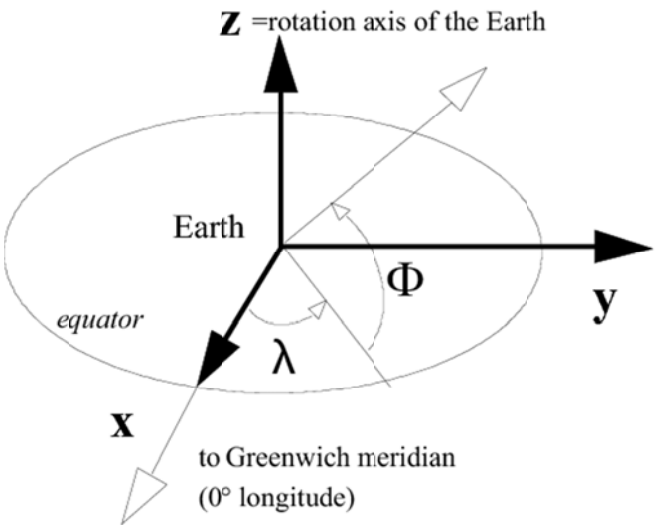

Figure 3.5 GEO System
The Z-axis is parallel to the rotation axis of the Earth. The X and Y axis are included in the equator plane. The X axis is pointing from the centre of the Earth to the Greenwich meridian (0° longitude). The *GEO* system is fixed with the rotating Earth. Longitude λ and latitude Φ are defined in this system in the same way as right ascension and declination in *GEI* system. This is an intermediate coordinate system required for some transformations.

3.4.1.5 Geocentric Equatorial Inertial System (GEI) or Earth-Centered Inertial (ECI) System

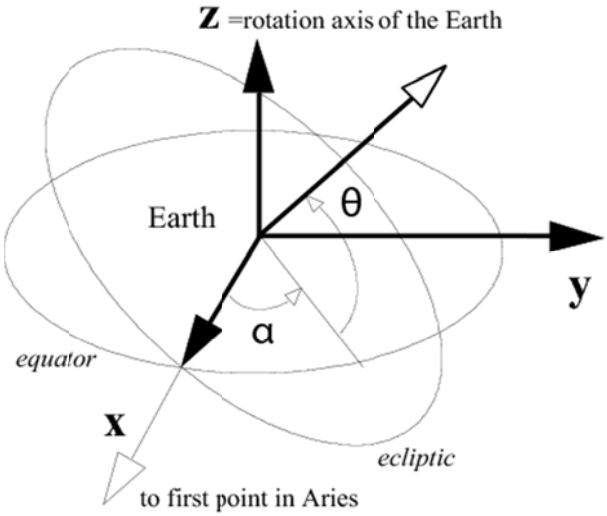

Figure 3.6 GEI or ECI System
The Z-axis is parallel to the rotation axis of the Earth. The X-axis is defined by the intersection of the equator plane and the ecliptic plane, and is pointing towards the first point of Aries (Sun position at the vernal equinox). The angles are *right ascension,* α, and the *declination* θ.

3.4.1.6 Geocentric Solar Ecliptic (GSE) System

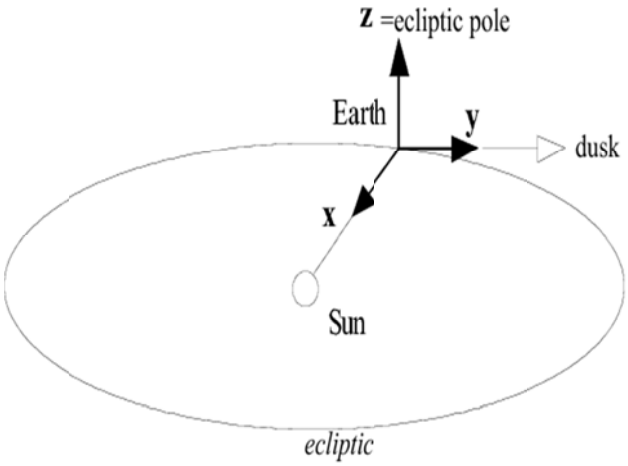

Figure 3.7 GSE System
The X-axis is pointing from the Earth towards the Sun. The X-axis and the Y-axis are included in the ecliptic plane. The Y-axis is pointing toward the dusk, opposing to the planetary motion. The Z-axis is parallel to the ecliptic pole. The *GSE* system has a yearly rotation with respect to the inertial system.

3.4.1.7 Geocentric Solar Magnetospheric (GSM) System

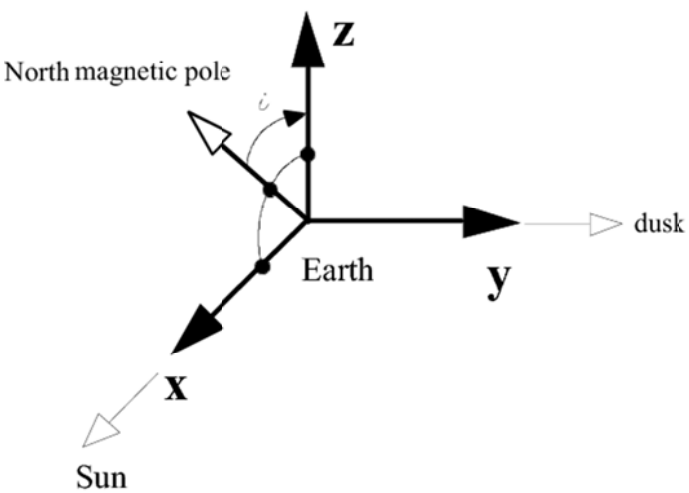

Figure 3.8 GSM System
The X-axis is pointing from the Earth towards the Sun. The X-Z plane contains the dipole axis. The Y-axis is perpendicular to the Earth's magnetic dipole, towards the dusk and include in the magnetic equator plane. The positive Z-axis is chosen to be in the same sense as the northern magnetic pole; the dipole tilt angle ***i*** is positive when the north magnetic pole is tilted towards the Sun. In addition to a yearly period due to the motion of the Earth about the Sun, the *GSM* system rocks about the solar direction with a 24 h period.

Solar Magnetic (SM) system

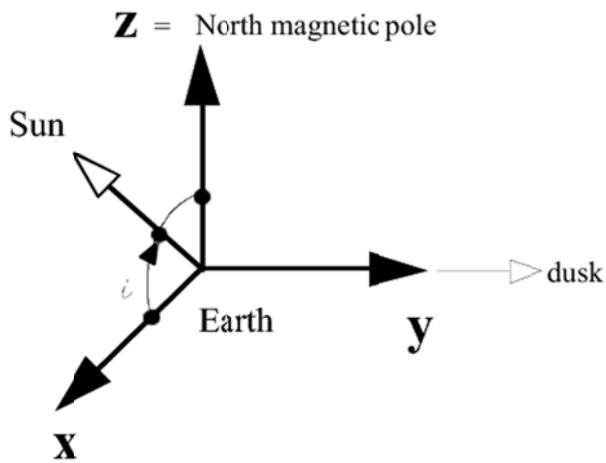

Figure 3.9: The Z-axis is parallel to the North magnetic pole, the X-Z plane contains the direction of the Sun and Y-axis is perpendicular to the Earth-Sun line toward dusk. The SM system rotates with both a yearly and daily period with respect the inertial system.

3.4.2 Mathematical Description

The transformations involve rotations only from one coordinate system to another. However, in order to carry out coordinate transformations we must calculate the various time-dependent rotation angles used in the transformations. The formulae used in the transformations provide a level of precision of 0.001 degrees up to the year 2100 for each rotation angle, which is deemed sufficient for GOES-R MAG data analysis applications.

3.4.2.1 Time and Date

The time used for all transformations is referenced to J2000 (12:00 UT on 1 January 2000). However, GOES data before J2000 can still use this reference date.

The Julian date (J_D) can be calculated given a Gregorian date using standard algorithm written by Robin O'Leary (http://pdc.ro.nu/jd-code.html)

y=y+8000
If month less than 3 then y=y-1 and m=m+12
J_D = (y*365) +(y/4) -(y/100) +(y/400) -1200820+(m*153+3)/5-92+d-1

$J_D = J_D$ +h/24.0+mi/1440.0+s/86400.0-0.5

where, y=year, m=month, d=day, h=hour, mi=minutes and s=seconds. The first line limits the calculation to only positive year values. Any value divisible by 4000 could be chosen, since the pattern of leap years in the reformed Gregorian calendar repeats every 4000 years. For the second line, since leap days are added at the end of February, it makes sense to consider January and February as the last two months (numbered 13 and 14) of the previous year. That way, leap days are always added at the end of the notional year. All results of calculations on the third line are integer only (i.e., no fractions), while the fourth line takes into account hours, minutes and seconds.

The number of Julian days past J2000 is given by

$$d_0 = \mathrm{J_D} - 2451545 .$$

The time can be converted to seconds past J2000 using

$$es = d_0 \cdot 86400 .$$

The seconds (es) are then converted into Julian centuries (T_0) using

T_0= (es / 86400.0) /36525.0

which, takes into account hours, minutes and seconds.

Since Julian date begins at noon it is of interest to calculate the time, *H*, in hours, since preceding UT midnight,

$$H = \left[\left(J_D - 0.5\right) - floor\left(J_D - 0.5\right)\right] \cdot 24.0$$
$$if\ \left(H < 0\right), H = H + 24.0$$

For dipole location estimation the decimal year (sdate) is required. We used the following algorithm:

F1 = (int)((275.0 * month) / 9.0)

F2 = (int)((month + 9.0) / 12.0)

N = F1 - (2 * F2) + day – 30
If leap year,

N = F1 - F2 + day – 30.

dblDay = N + (hour + (min + (sec / 60.0)) / 60.0) / 24.0;

sdate = year + dblDay/366 (1-365.999999... day range

if leap year,

sdate year+dblDay/367 (1-366.999999... day range)

3.4.2.2 Earth Magnetic Dipole Location

The geographic position of the Earth magnetic pole can be calculated from the interpolated first three coefficients g_{10}, g_{11} and h_{11} of the IGRF2005. The geographic longitude λ_D and latitude Φ_D is found using the formulae

$$\lambda_D = \arctan(h_{11} / g_{11})$$

$$\Phi_D = 90^o - \arctan \frac{g_{11} \cos \lambda_D + h_{11} \sin \lambda_D}{g_{10}}$$

where, λ_D lies in the fourth quadrant. The coefficients can be found online at http://www.ngdc.noaa.gov/IAGA/vmod/igrf.html.

In order to interpolate the coefficients we create an array containing the IGRF epoch years. Interpolation of the coefficients is accomplished by the following:

- The decimal year (sdate) is calculated.
- An array containing the epoch IGRF years is searched:
  - Search for epochs < sdate, while the_model++.
  - the_model = epoch year index.
  - Fraction of years since epoch year is given by
    - fracYear=(sdate-epochs[the_model-1])/(epochs[the_model]-epochs[the_model-1]).
- Interpolated coefficient values are calculated using the equation (e.g., g01):
  - g01[the_model-1]+fracYear*(g01[the_model]-g01[the_model-1])

3.4.2.3 Greenwich Mean Sidereal Time

The Greenwich mean sidereal time (θ_{GMST}) in degrees can be calculated using

$$\theta_{GMST} = 280.46061837 + 360.98564736629 \cdot d_0 + 0.0003875 \cdot T_0^{\ 2} - 0.0000000258333 \cdot T_0^3$$

where, d_0 is Julian days from J2000 and T_0 is Julian centuries from J2000. The formula is given by M. Franz and D. Harper (see, http://www.space-plasma.qmul.ac.uk/heliocoords/systems2art/).

An alternative calculation for the GMST angle is

$$\theta_{GMST} = 100.461 + 360000.770 \cdot T_0 + 15.0410 \cdot H \ \text{ degrees}$$

where, T_0 is the time in Julian centuries from 12:00 UT on 1 January 2000 to the midnight Universal Time (UT) preceding the time of interest and H is the time in hours since that preceding UT midnight. Formula derived from the Almanac for Computers.

3.4.2.4 Sun Location

Calculation of the Sun location, which is required for conversion to GEI, GSE and GSM coordinates are based on J2000 epoch. The Sun's ecliptic longitude (λ_s) can be calculated using the series of formulae:

$$M = 357.528 + 35999.050T_0 + 0.04107 \cdot H \ \text{ degrees}$$
$$\lambda_0 = 280.460 + 36000.772 \cdot T_0 + 0.04107 \cdot H \ \text{ degrees}$$
$$\lambda_s = \lambda_0 + (1.915 - 0.0048 \cdot T_0) \cdot \sin M + 0.020 \cdot \sin 2M \ \text{ degrees}$$

where, T_0 is the time in Julian centuries from 12:00 UT on 1 January 2000 to the midnight Universal Time (UT) preceding the time of interest and H is the time in hours since that preceding UT midnight. Formulae derived from the Almanac for Computers. In the intermediate formulae, M is the Sun's mean anomaly and λ_0 its mean longitude.

The obliquity of the ecliptic (ε) can be calculated using the formula:

$$\varepsilon = 23.439 - 0.013 \cdot T_0 \text{ degrees}$$

where, T_0 is the time in Julian centuries from 12:00 UT on 1 January 2000 to the midnight (UT) preceding the time of interest. Formula derived from the Almanac for Computers.

3.4.2.7 Coordinate Transformations

In order to easily describe the different transformations we use a notation that follows the approach of Hapgood (1992) in which complex transformations are factorised into a series of simple rotations about principal axes (i.e. the cartesian axes X, Y or Z). Only two parameters are required to specify the matrix for a simple rotation:

- the rotation angle ζ, and
- the name of the rotation axis (X, Y or Z).

We use the notation

$$E = \langle \zeta, axis \rangle$$

to denote the matrix for a simple rotation, where ζ is the rotation angle and *axis* is the name of the rotation axis (X, Y or Z). The matrices are

$$\langle \zeta, X \rangle = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\zeta & \sin\zeta \\ 0 & -\sin\zeta & \cos\zeta \end{bmatrix}$$

$$\langle \zeta, Y \rangle = \begin{bmatrix} \cos\zeta & 0 & \sin\zeta \\ 0 & 1 & 0 \\ -\sin\zeta & 0 & \cos\zeta \end{bmatrix}$$

$$\langle \zeta, Z \rangle = \begin{bmatrix} \cos\zeta & \sin\zeta & 0 \\ -\sin\zeta & \cos\zeta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For inverse rotation the notation is

$E = \langle -\zeta, axis \rangle$,

while the product matrix is

$T = E_1 \cdot E_2 = \langle \zeta_1, axis_1 \rangle \cdot \langle \zeta_2, axis_2 \rangle$

Coordinate transformation is accomplished by the multiplication

$V' = T \cdot V$

where, *V* is the column vector representing the three components of the MAG in the old coordinates and *V'* is another column vector representing components of the MAG in the new coordinate system. *T* is the 3x3 matrix containing the transformation rotational coefficients.

The coordinate transformations used in the algorithms are:

1. The ***GEO to GLS*** transformation is given by the matrix

$$T_3 = \langle \lambda \ , Z \rangle$$

where, λ is the longitude of GOES in geographic coordinates. For GLS to GEO we are actually interested in the inverse transformation

$$T_3^{-1} = \langle -\lambda \ , Z \rangle$$

which, takes *GLS* coordinate data to the intermediate coordinate *GEO* system for transformation to the *GEI* coordinate system.

2. The ***GLS to VDH*** transformation

In order to rotate from GLS to VDH we require knowledge of the dipole axis in GLS coordinates. We define the unit vector H describing the direction of the geomagnetic dipole axis in the GLS coordinate system, which can be calculated as follows:

a. Define the unit vector describing the direction of the geomagnetic dipole axis in GEO coordinate system as

$$Q_g = \begin{pmatrix} \cos\Phi_D \cos\lambda_D \\ \cos\Phi_D \sin\lambda_D \\ \sin\Phi_D \end{pmatrix}$$

where, Φ_D and λ_D are the GEO latitude and longitude of the geomagnetic pole as derived above.

b. Apply the matrix formula

$$H = T_3 \cdot Q_g$$

where, T_3 is the GEO to GLS transformation matrix.

To find the *D*-component of the *VDH* system we first define the spacecraft location unit vector in GLS coordinates as

$$Rg = \begin{pmatrix} \cos\Phi g \\ 0 \\ \sin\Phi g \end{pmatrix}$$

where, Φ_D is the latitude of the GOES spacecraft. The eastward component is then given by

$$D = \frac{H \times R_g}{\left| H \times R_g \right|}.$$

The radial component is

$$V = D \times H$$

which completes the right hand system. Hence, the total GLS to VDH coordinate transformation matrix is

$$T_7 = \begin{bmatrix} V(0) & V(1) & V(2) \\ D(0) & D(1) & D(2) \\ H(0) & H(1) & H(2) \end{bmatrix}$$

3. The ***GEI to GEO*** transformation is given by the matrix

$$T_4 = \langle \theta_{GMST}, Z \rangle$$

where, the rotation angle θ_{GMST} is the Greenwich mean sidereal time. This transformation is a rotation in the plane of the Earth's equator from the First Point of Aries to the Greenwich meridian. However, for GEO to GEI we are interested in the reverse transformation

$$T_4^{-1} = \langle -\theta_{GMST}, Z \rangle$$

4. The ***GEI to GSE*** transformation is given by the matrix

$$T_5 = \langle \lambda_s, Z \rangle \cdot \langle \varepsilon, X \rangle,$$

where, the rotation angle λ_s is the Sun's ecliptic longitude and the angle ε is the obliquity of the ecliptic. This transformation is a rotation from the Earth's equator to the plane of the ecliptic followed by a rotation in the plane of the ecliptic from the First Point of Aries to the Earth-Sun direction.

5. The The ***GSE to GSM*** transformation

The dipole location is generally given in geographic coordinates; however, the geographic equator is tilted with respect to the ecliptic. The GSM dipole tilt (μ) and the GSE-GSM angle (ψ) can be calculated using the formulae:

$$\mu = \tan^{-1}\left(\frac{x_e}{\sqrt{y_e^2 + z_e^2}} \right)$$

$$\psi = \tan^{-1}\left(\frac{y_e}{z_e} \right)$$

where, x_e, y_e and z_e are the components of a unit vector Q_e describing the direction of the geomagnetic dipole axis in the GSE coordinate system. We can calculate Q_e as follows:

a. Define the unit vector describing the direction of the geomagnetic dipole axis in the GEO coordinate system, given by

$$Q_g = \begin{pmatrix} \cos\Phi_D \cos\lambda_D \\ \cos\Phi_D \sin\lambda_D \\ \sin\Phi_D \end{pmatrix}$$

where, Φ_D and λ_D are the GEO latitude and longitude of the geomagnetic pole as derived above.

b. Apply the matrix formula:

$$Q_e = T_5 \cdot T_4^{-1} \cdot Q_g$$

where, T_4 and T_5 are the ***GEI to GEO*** and ***GEI to GSE*** transformation matrices, respectively.

The GSE to GSM transformation matrix is given by

$T_6 = \langle -\psi, X \rangle$,

where, the rotation angle ψ is the *GSE-GSM* angle. This transformation is a rotation in the *GSE* YZ plane from the *GSE* Z axis to the *GSM* Z axis.

6. The ***GSM to SM*** transformation is given by the matrix

$T_8 = \langle -\mu, X \rangle$,

where, the rotation angle μ is the dipole tilt defined in the GSE to GSM transformation. This transformation is a rotation in the GSM XZ plane from the GSM Z axis to the geomagnetic dipole axis.

3.4.2.8 Computation of transformation Matrix Elements

For computational considerations we calculate the nine terms in each simple rotation matrix as follows:

1. calculate $\sin\zeta$ and $\cos\zeta$,
2. determine the matrix diagonal:
    1. put 1 in the *N*th term, where *N*=1 if the rotation axis is X, *N*=2 if Y and *N*=3 if Z,
    2. put $\cos\zeta$ in the other two terms
3. locate the two off-diagonal terms in the same columns and rows as the $\cos\zeta$ terms - put $\sin\zeta$ in the term above the diagonal and $-\sin\zeta$ in the term below,
4. set the remaining off-diagonal terms to zero.

The signs of the $\sin\zeta$ terms above define the sense of positive rotation in this approach. When more than one simple rotation is required each simple rotation is first defined as described above and then the simple rotations a multiplied to give the final transformation matrix, *T*.

3.4.3 Algorithm Output

The algorithm processes data assuming an input data stream and outputs a data stream. Each three data sample (X,Y,Z) is converted into one or more of the four alternative coordinate systems (VDH, GEI, GSE and GSM) and the results returned (outputted) as ASCII.

4.0 TEST DATA SETS AND OUTPUTS

4.1 Simulated/Proxy Input Data Sets

Proxy input data sets are taken from previous GOES MAG measurements. GOES MAG data can be downloaded using the FRODO IDL software package at SWPC and the NASA NSSDC website http://nssdc.gsfc.nasa.gov/. In addition, time intervals published by the space physics community were chosen.

There was no simulated data generated to test the algorithm.

We tested coordinate transformation to GSE, GSM and VDH. There was no testing of GEI (ECI), GEO or GSL coordinates since they are intermediate steps during conversion to at least one of the tested transfromations.

4.2 Output from Simulated/Proxy Inputs Data Sets

GEI to VDH

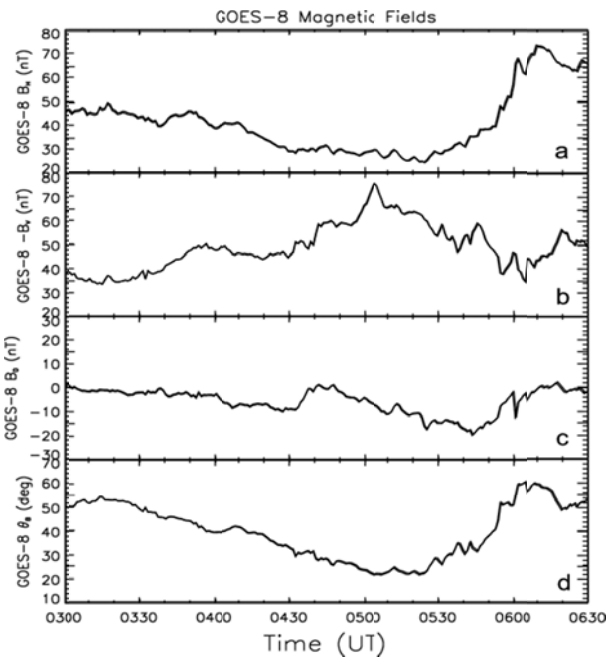

Figure 4.1 Comparison of Liang et al (2004) VDH calculation to GOES-R VDH algorithm. The comparisons are perfect.

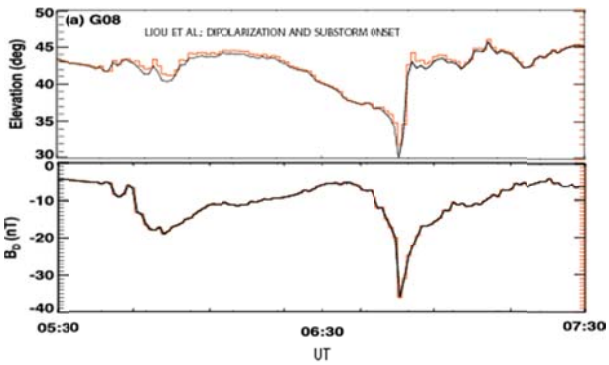

Figure 4.2 Comparison of Liou et al (2002) VDH calculation (pink line) to GOES-R VDH algorithm (black).

GEI to GSE

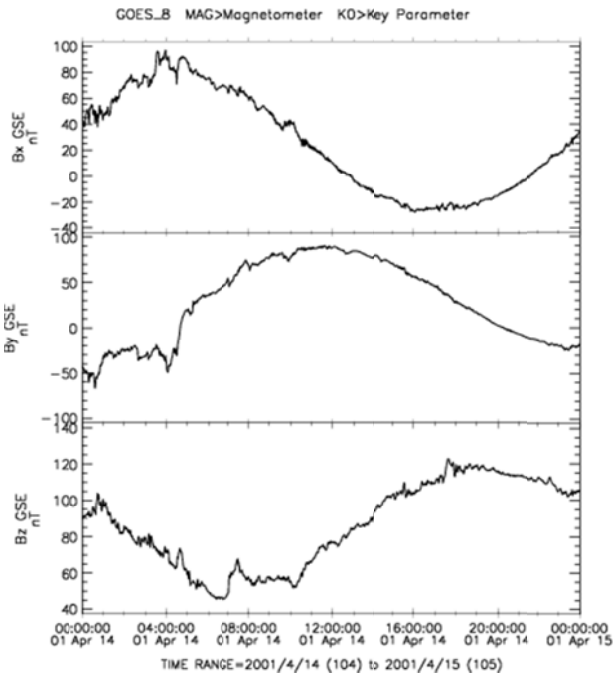

Figure 4.3: Comparison of legacy GOES GSE calculation to GOES-R GSE algorithm. Comparison is perfect.

GEI to GSM

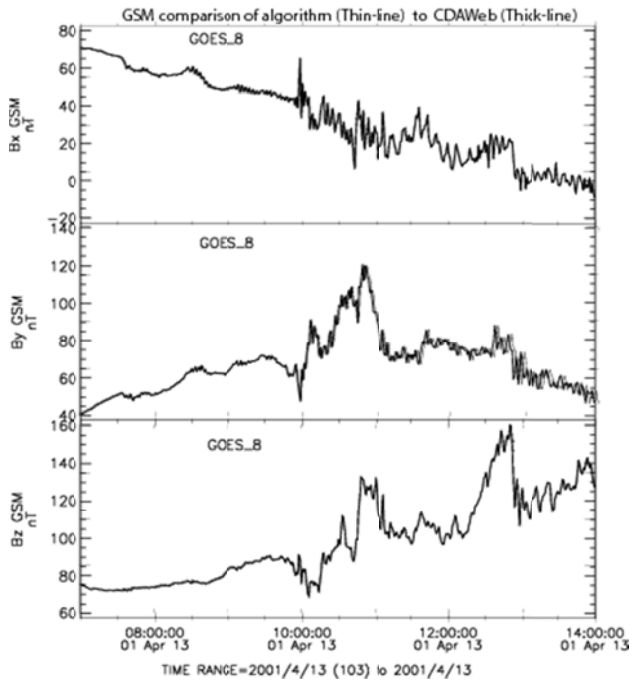

Figure 4.5: Comparison of legacy GOES GSM calculation to GOES-R GSM algorithm. Comparison is perfect.

4.2.1 Precisions and Accuracy Estimates

The formulae used in the transformations mostly follow the workd of Hapgood (1992; 1995; 1997) and references within. They provide a level of precision of 0.001 degrees up to the year 2100 for each rotation angle, which is deemed sufficient for GOES-R MAG data analysis applications.

4.2.2 Error Budget

Not applicable.

5.0 PRACTICAL CONSIDERATIONS

5.1 Numerical Computation Considerations

Computation time is not a factor.

5.2 Programming and Procedural Considerations

The dipole calculation requires year fraction while transformations require seconds past J2000. Hence, separate objects handle dipole and transformation calculations. The transformations are calculated sequencially using member functions. For example, GEI to GSM is implmeneted by

gei to gse transformation followed by
gse to gsm transformation

5.3 Quality Assessment and Diagnostics

The following procedures are recommended for diagnosing the performance of the GOES-R MAG coordinate transformations.

- Routinely compare transtormation results to research community publications.

5.4 Exception Handling

Transformation member functions return 0 or 1. 0 means no error and 1 means error. However, no exception action is taken given an error. Exception handling will have to be included in future updates. The algorithm does not consider invalid input data, hence, invalid values must be removed/filled in after transformation.

5.5 Algorithm Validation

Pre-Launch Validation:

The algorithm will be/has been validated as described in section 4 using previous GOES magnetometer data as proxy and comparisons to research community results.

Post-Launch Validation:

The algorithm will be validated using high resolution (8 Hz) actual flight level 1B data and comparisons to research community results.

6.0 ASSUMPTIONS AND LIMITATIONS

6.1 Performance

The formulae used in the transformations mostly follow the work of Hapgood (1992; 1995; 1997) and references within. They provide a level of precision of 0.001 degrees up to the year 2100 for each rotation angle, which is deemed sufficient for GOES-R MAG data analysis applications. However, updates to IGRF model will be required every five years.

6.2 Assumed Sensor Performance

The algorithm assumes the sensor will meet its specifications and performance is within normal operational parameters. When invalid sensor data occurs, it is assumed that these errors are replaced by error fill in values, e.g. 9999, after transformations.

6.3 Pre-Planned Product Improvements

To facilitate comparisons with older GOES data ENP to ECI transformation has been developed and will be implemented.

Plan to improve consistency across algorithm objects by re-writing the dipole algorithm to use the same date/time format as transformation objects.

7.0 REFERENCES

(1) C.T. Russell, (1971) "Geophysical Coordinate Transformations", *Cosmic. Electrodyn.* **2**, 184-196.

(2) M.A. Hapgood, (1992) "Space Physics Coordinate Transformations: A User Guide", *Planet. Space Sci.* **40**, 711-717.

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(4) M.A. Hapgood, (1997) "Corrigendum to Space Physics Coordinate Transformations: A User Guide", *Planet. Space Sci.* **45**, 1047.