REPORT UAG-68

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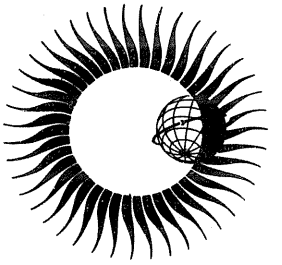
# Solar-Terrestrial Physics

# A COMPARATIVE STUDY OF METHODS OF ELECTRON DENSITY PROFILE ANALYSIS

Chairman's Report, URSI Subgroup G/6/2 on N(h) Analysis



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Chairman's Report, URSI Subgroup G/6/2 on N(h) Analysis


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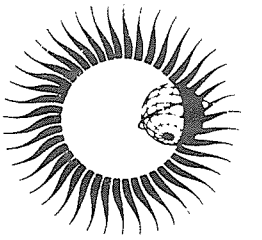
SEPTEMBER 1978

Published by World Data Center A for
Solar-Terrestrial Physics, NOAA, Boulder, Colorado
and printed by

U.S. DEPARTMENT OF COMMERCE
NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
ENVIRONMENTAL DATA AND INFORMATION SERVICE
Asheville, North Carolina, USA 28801


SUBSCRIPTION PRICE: $25.20 a year; $17.30 additional for foreign mailing; single copy price varies.* Checks and money orders should be made payable to the Department of Commerce, NOAA/NGSDC. Remittance and correspondence regarding subscriptions should be sent to the National Geophysical and Solar–Terrestrial Data Center, NOAA, Boulder, CO 80303.

PRICE THIS ISSUE $1.41

TABLE OF CONTENTS


| | | Page |
|---|---|---|
| ABSTRACT | | 1 |
| 1. | INTRODUCTION | 1 |
| 2. | THE PROBLEMS IN N(h) ANALYSIS | 2 |
| | 2.1 The Starting Problem | 2 |
| | 2.2 The Valley Problem | 2 |
| 3. | ANALYSIS OF NUMERICAL IONOGRAMS | 3 |
| | 3.1 Complete Monotonic Profiles | 3 |
| | 3.2 The Starting Problem | 4 |
| | 3.3 The Valley Problem | 4 |
| 4. | SIMPLE METHODS OF ANALYSIS | 5 |
| 5. | CONCLUSION | 6 |
| 6. | TOPICS FOR FURTHER STUDY | 7 |
| 7. | REFERENCES | 8 |

TABLE

| | | |
|---|---|---|
| 1 | The Real Height Errors (Calculated Value - Correct Value), in Meters, Obtained Using 10 Different Methods of Analysis | 9 |

FIGURES

| | | |
|---|---|---|
| 1 | General Overview of the Aspects of N(h) Analysis of Ionograms | 10 |
| 2 | Calculated and Correct Real Heights for the Lower Parts of Profile 1 Used for the Underlying Ionization Tests | 11 |
| 3 | Calculated and Correct Real Heights for the Lower Parts of Profile 1 Used for the Underlying Ionization Tests | 12 |
| 4 | Calculated and Correct Real Heights for the Lower Parts of Profile 2 Used for the Underlying Ionization Tests | 13 |
| 5 | Calculated and Correct Real Heights for the Lower Parts of Profile 2 Used for the Underlying Ionization Tests | 14 |
| 6 | Calculated and Correct Real Heights for the Lower Parts of Profile 3 Used for the Underlying Ionization Tests | 15 |
| 7 | Calculated and Correct Real Heights for the Lower Parts of Profile 4 Used for the Underlying Ionization Tests | 16 |
| 8 | Calculated and Correct Real Heights for the Lower Parts of Profile 5 Used for the Underlying Ionization Tests | 17 |
| 9 | Calculated and Correct Real Heights for the Lower Parts of Profile 6 Used for the Underlying Ionization Tests | 18 |
| 10 | Calculated and Correct Real Heights for the Base of the F Layer for the Profile 1 Used for the Valley Tests | 19 |
| 11 | Calculated and Correct Real Heights for the Base of the F Layer for the Profile 2 Used for the Valley Tests | 20 |
| 12 | Calculated and Correct Real Heights for the Base of the F Layer for the Profile 3 Used for the Valley Tests | 21 |
| 13 | Calculated and Correct Real Heights for the Base of the F Layer for the Profile 4 Used for the Valley Tests | 22 |
| 14 | Calculated and Correct Real Heights for the Base of the F Layer for the Profile 5 Used for the Valley Tests | 23 |
| 15 | Calculated and Correct Real Heights for the Base of the F Layer for the Profile 6 Used for the Valley Tests | 24 |

APPENDICES

| | | |
|---|---|---|
| A | N(h) Analysis - Historical Survey and Summary of Results by J. E. Titheridge | 25 |
| B | Summary of N(h) Analysis Approach by T. L. Gulyaeva | 37 |
| C | Conversion Methods for h'(f) to N(h) Used at Lindau by W. Becker | 41 |
| D | Historical Summary by Adolf K. Paul | 46 |
| E | Historical Summary by J. W. Wright | 48 |
| F | Bibliography of N(h) Profile Analysis by L. F. McNamara | 51 |
| G | Address List of Subgroup G/6/2 Members | 56 |

# A COMPARATIVE STUDY OF METHODS OF ELECTRON DENSITY PROFILE ANALYSIS
## Chairman's Report, URSI Subgroup G/6/2 on N(h) Analysis

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ABSTRACT. URSI Subgroup G/6/2 was charged with comparing available methods of real-height analysis of ionograms in order to determine the best method. Experts in the field who composed the subgroup attempted to compare all known available methods. The comparisons were made on the basis of the accuracy of the results obtained by each method in the analyses of a series of accurate numerical ionograms, with the correct results being known only to the chairman. Of those methods for which sufficient results were presented to allow definitive conclusions to be drawn, those of A. K. Paul (Boulder, Colorado, U.S.A.) and J. E. Titheridge (Auckland, N.Z.) proved to be the most promising.

## 1. INTRODUCTION

Subgroup G/6/2 of Working Group 6, Commission G of the International Union of Radio Science (URSI), was formed after the 17th URSI General Assembly in Warsaw, 1972. Subgroup G/6/2 was asked to compare extant methods of real-height analysis of ionograms in order to recommend the "best" technique(s).

Mr. C. G. McCue (Director of the Australian Ionospheric Prediction Service) was appointed chairman of the subgroup in early 1973 and immediately invited prominent workers in the field to participate in the endeavor. The nominal list of members prior to its reorganization at Helsinki is given in Appendix G, although, as will be seen later, the effective list was much smaller. The author and present chairman, L. F. McNamara, acted as secretary under McCue's chairmanship.

Between its inception and the 18th URSI General Assembly in Lima, 1975, the subgroup's main activity was the analysis of representative numerical ionograms in order to identify those methods that coped best with the special problems encountered in this field. During a meeting in Lima, however, several subgroup members expressed their dissatisfaction with the comparative tests that had been used. They recommended that a new set be conducted with different guidelines and be organized by the new chairman, L. F. McNamara.

This report describes the new set of tests -- those dating from 1975 -- and the results obtained. It also shows how the field has developed over the past 10 or so years since the Radio Science Special Issue on Analysis of Ionograms for Electron Density Profiles was published in October 1967 [Wright and Smith, 1967].

The different methods of analysis considered are discussed here mainly in terms of their accuracy as indicated by their results for the test ionograms. Other important aspects of the problem, as far as the use of the methods by persons other than the originators is concerned, are the availability of the computer program, the ease with which the program may be applied routinely, and the computation time for each complete analysis. These topics were discussed at a meeting of the subgroup in Helsinki, preceding the 19th URSI General Assembly. A general overview of the problem is given in Figure 1.

Methods of analysis discussed in this report are those used by L. Bossy [1978]; W. Becker, J. E. Titheridge, A. K. Paul, T. L. Gulyaeva (Appendices A-D of this report); and H. H. Howe and D. E. McKinnis [1967] as used by J. W. Wright (Appendix E of this report). Although the set of tests conducted before 1975 also included results for the methods of O. A. Mal'tseva, A. K. Saha, and J. E. Jackson as used by J. E. Bennett and J. S. Nisbet, these three techniques are not considered here because (1) the Mal'tseva and Jackson methods were found to be significantly less accurate for the pre-1975 tests than the former set of methods, (2) Saha did not provide results for the second set of tests (partly because he felt they were unrealistic), and (3) Saha provided results for only the monotonic profile in the first set. Bossy provided results for only the monotonic profile of the second set of tests. Becker submitted results for this profile and partial results for the "starting" problem test.

Section 2 of this report gives a brief introduction to the problems encountered in N(h) analysis, addressing them by the analysis of test ionograms described in Section 3.

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Conclusions are given in Section 5. This section and Section 6 were prepared following discussions of the subgroup at Helsinki.

Appendices A, B, C, D and E give historical surveys of the development of the methods used by members who have completed all tests, or who intend to do so. These surveys are designed to help the non-expert through the literature, showing how each member's ideas have evolved, and the context of each publication. They have been written by the individual members, without criticisms by other members and may possibly contain personal opinions that are not generally accepted and that may become subjects of further study by the subgroup. Many important aspects of N(h) analysis are also discussed in the appendices, which can therefore be used to complement the main text.

A bibliography for the period 1969 (approximately) onwards is given in Appendix F.

## 2. THE PROBLEMS IN N(h) ANALYSIS

Even when very accurate virtual heights are available, there still exist two fundamental problems in real-height or N(h) analysis. These are referred to as the "starting" problem and the "valley" problem, and both are manifestations of incomplete data. A third difficulty, arising from a lack of data, is the "peak extrapolation" problem. This is discussed by Titheridge et al. [1978].

Errors in the ionograms themselves and in the extraction of virtual height data from the ionograms compound the problems mentioned above, in the analysis of practical ionograms. If individual profiles are being studied, it is desirable to have some estimate of the uncertainty in the calculated real heights, since if this is too large the results may be worthless. These topics are not discussed here, since the attentions of the subgroup were directed to comparative analyses of accurate virtual height data. This can be done only by excluding other factors which cause errors in the calculated heights.

### 2.1 The Starting Problem

With practical ionograms, the accuracy of a calculated N(h) profile may be severely restricted because reflections are not obtained below some limiting frequency $f_{min}$. This limit is set by choice, by broadcast station interference, by the output power of the ionosonde, by the antenna system, or by the rapid increase of absorption toward lower frequencies. Each of these effects leads to what is known as the "starting" problem in N(h) analysis.

For a given ordinary ray virtual height curve, extending down to the frequency $f_{min}$, there exists an infinite number of possible real height curves corresponding to different ionization distributions below $f_{min}$. If only ordinary ray data are available, the choice among alternatives must be made on physical grounds. Methods of analysis must therefore incorporate some assumptions about the profile in the "unseen" regions. These possible assumptions may be classified into six different groups, which have been critically discussed by Titheridge [1975]. Extraordinary ray measurements are essential for a mathematical solution to the problem. These measurements are not always available, however, so assumptions based on expediency, physical considerations, or experience must often be employed.

### 2.2 The Valley Problem

The "valley" or decrease of electron density between the E- and F-layers of the ionosphere has long been a problem in the analysis of ground-based ionograms. The simpler ionogram reduction techniques have generally ignored the valley, assuming that the electron density increases monotonically with height. This assumption, or the use of some assumed valley size, is necessary if only O-ray virtual data are available, since there is an infinite number of valleys that can satisfy the data. More sophisticated techniques involve analyzing the ionogram by normal monotonic methods up to the E-layer peak, followed by a special restart procedure.

The valley problem is essentially a daytime problem, since at night foE < $f_{min}$ for most ionograms, and any valley problem is subsumed by the starting problem. The techniques used to overcome the valley problem are usually quite similar to those used for the starting problem and depend on the use of X-ray data above fxE. In the absence of such data, it is necessary to assume that there is no valley (i.e., that the profile is monotonic) or to assume some synoptic value of the valley shape.

If in the absence of X-ray data the monotonic assumption is made, the calculated heights will always be too low if there is in fact a valley. In general, it seems preferable to introduce a valley of some simple shape with parameters in accordance with rocket and incoherent scatter observations. Tests of analysis methods in this case are rather pointless, because the errors depend on the fortuitous agreement between the actual and assumed valley profiles. It should be noted, however, that many methods include the option of using a synoptic valley in the absence of X-ray data.

## 3. ANALYSIS OF NUMERICAL IONOGRAMS

This section describes the results obtained by group members for the analysis of accurate numerical ionograms. Only a summary is given here, because the test ionograms and the results have been described in detail elsewhere [McNamara and Titheridge, 1977; Titheridge et al., 1978; Gulyaeva et al., 1978; Paul et al., 1978].

### 3.1 Complete Monotonic Profiles

The ionograms analyzed corresponded to a monotonic profile composed of two overlapping Chapman layers. Virtual heights accurate to 0.001 km were given for a dip angle of 65°. Ionograms were given for a constant gyrofrequency of 1.2 MHz, and for a gyrofrequency of 1.2 MHz at the ground and decreasing as the inverse cube of the height. The latter corresponds more closely with the real situation. The model profile was cut off at $f_{min}$, with $h_1 = h'_1$, to avoid any starting problem.

The results of the analyses are given in detail in Titheridge et al. [1978]. Table 1 displays the real height errors (calculated value - correct value) obtained at selected points for the constant gyrofrequency case using the different methods. The values of frequency chosen correspond to neither a given data point nor to the midpoint of any lamina, in order to identify methods yielding large errors between data points. In fact none of the methods considered did so.

Titheridge provided results for his linear lamination method, LINAN [1978a], his overlapping polynomial method, LAPOL [1967a + improvements], and his generalized polynomial method, POLAN [1978b]. For POLAN, results were provided for modes 3, 4, and 5, corresponding to the use of somewhat different polynomial techniques. Other members used the methods described in the appendices. Gulyaeva also provided results for the case in which she permits the first derivative to be discontinuous at the cusp (Column 9 in Table 1).

Table 1 also lists the numbers of data points used by each member along with the smallest frequency interval near the cusp of foE. The number of data points necessary to restrict the analysis errors to below an acceptable level is an important practical consideration. Clearly, the smaller the number the better, since a smaller number leads to less time spent scaling the ionogram and less computer time spent in analyzing the scaled data. This problem will not be examined at length here.

The "Method" row of Table 1 indicates whether the calculated heights were obtained using the assumed profile variation between two adjacent data points (F) or by some form of interpolation in the array of calculated heights (I). For arrays $f_i$, $h_i$, $i = 1$ to $n$ with the peak at $i = n$, Titheridge used second difference interpolation to get $h(f)$ when $f < f_{n-2}$ and inverse interpolation (taking $f$ as a function of $h$) when $f > f_{n-2}$.

In general, it would be expected that the errors would be largest near foE, depending on the sharpness of the cusp, and near foF2, depending on the amount of curvature of the F-layer peak.

In this particular case the cusp is not very sharp, as with real ionograms, so the errors remain relatively small, i.e., $\lesssim$ 0.1 km. For a much sharper cusp and using a frequency interval of 0.1 MHz, errors of 0.5-1.0 km are typical. The problem is then essentially one of under-sampling, however, and can be overcome by the use of a smaller frequency interval ($\lesssim$ 0.02 MHz), although such small intervals are not normally used in practice.

The expected increase in error toward foF2 is not well illustrated by Table 1, apart from Becker's results. In general, the error will be greatest for those methods using the simplest form for the variation between scaled frequencies (excluding Titheridge's LINAN, which uses off-set frequencies). The error again depends on the interval between scaled frequencies and can be reduced to a negligible size by decreasing the interval to $\lesssim$ 0.1 MHz.

The error at the highest frequency in Table 1, 6.985 MHz, also depends on the accuracy of the peak extrapolation procedure, if this extrapolation was commenced at a frequency below 6.985 MHz. This is the case for Paul, Gulyaeva and Titheridge, who used 6.950, 6.980 and 6.980 MHz, respectively, as their highest frequency. Becker used a highest frequency of 6.810 MHz, so his errors at the highest two frequencies are affected.

It can be seen from Table 1 that the analysis errors are very small for all methods considered, although they tend to become larger near the F2 peak. The smallest RMS errors were obtained by Titheridge (POLAN, mode 4), but in terms of an acceptable level of error, all methods are quite adequate. The errors for all methods can be made negligibly small simply by increasing the number of data points.

In a practical situation these analysis errors would be compounded with and largely out-weighed by uncertainties due to possible and largely irreducible errors in the input data.

Titheridge et al. [1978] also discuss the following topics which are important facets of the N(h) analysis problem:

1. Relative strengths of parabolic and polynomial methods;
2. Choice of scaling frequencies;
3. Peak extrapolation procedures;
4. Height-dependent gyrofrequency;
5. The single polynomial method;
6. Errors in the input data, including random scaling errors, zero-height error and receiver delay.

### 3.2 The Starting Problem

The ionograms analyzed have been described in detail by McNamara and Titheridge [1977, Section 3]. Because no guarantee exists that the success of any analysis method that relies on O-ray data only is not just fortuitous, X-ray data were provided in all cases.

Numerical ionograms, corresponding to six incoherent scatter profiles obtained at Arecibo, have been analyzed. Profiles 1 and 2 are analytic approximations to typical observed profiles for the magnetically quiet night of 12 April 1972. They correspond approximately to mean profiles between 0200 - 0500 LT and between 2100 - 2400 LT, respectively, but both shapes appear at different times before 2100 LT. Profile 2 has a considerably greater amount of underlying ionization and a more sharply bounded reflection layer than does profile 1. Profiles 3-6 are individual, April and July observed profiles (averaged over 5 min) in which the assumed density variation between data points is linear up to some height H in the base of the F layer. Above this height, the F-layer density varies linearly with $\ell n(N)$, closely matching the observed F-layer profile near its base.

As well as including six different underlying ionization distributions, this set of profiles contains two kinds of F-layer density distributions, namely hyperbolic ones (1 and 2) and ones linear in $\ell n(N)$ (3 - 6). Thus, although a few profiles favored some methods of analysis, no technique was favored consistently. It should be noted, however, that Titheridge's method is the only one which allows for a varying curvature in the reflecting region, which is likely to be the case in practice. The other methods are based on the assumption that there are no variations higher than parabolic.

A total of 24 analyses was requested, corresponding to six models, two dip angles and two values for $f_{min}$. The frequency spacing of the data was 0.02 MHz, and values of $h'_x$ greater than 500 km were not provided (for profile 2) since such values would not be expected in practice. Data were given for dip angles of 30° (with $f_H$ = 1.0 MHz) and 75° (with $f_H$ = 1.5 MHz) in order to test for any dependence of the errors on these parameters, with $f_H$ being an inverse-cube function of height.

Authors were requested to perform the analyses for two values of $f_{min}$, either 0.8 or 0.9 MHz and 1.5 MHz, the latter being a more likely observed value.

Details of the results are given in Gulyaeva et al. [1978]. Figures 2-9 show the correct and calculated real heights for the six models. In general, the largest errors were obtained for profile 2 at a dip angle of 30° (see Figure 4).

The right-hand axes or insets to figures 2-9 give the results for an $f_{min}$ of 1.5 MHz, a typical value for that parameter. There does not seem to be any marked change in the accuracy of the results of the different methods, although Titheridge's program POLAN did yield smaller errors for profile 2 at a dip angle of 75° (see Figure 5).

There does not seem to be any consistent dependence of the uncertainty in the calculated real heights on the dip angle. The effect of the height-variation of the gyrofrequency was found to be negligible.

A figure of ±5 to 10 km seems an appropriate estimate of the maximum error that should be anticipated when accurate O- and X-ray data are available. Experimental errors will tend to increase the uncertainty in the calculated profile, but their effects will be somewhat mitigated by use of least-squares techniques in the analyses. On the average, the error can be expected to be about a factor of four lower than the error introduced by failing to correct for the underlying ionization, but in some cases the decrease will be much larger. It would be difficult to estimate the uncertainty of results based on only O-ray data, but in general, the errors would be expected to exceed those obtained using X-ray data as well.

### 3.3 The Valley Problem

The ionograms analyzed have been described in detail by McNamara and Titheridge [1977, Section 4]. X-ray data have been provided in all cases.

Profiles 1, 2, and 3 are composed of two overlapping Chapman layers as described in McNamara and Titheridge [1977]. Profiles 1 and 2 correspond to midday conditions and have small valleys with widths of ∿5 and ∿16 km, respectively. Profile 3 corresponds to sunset conditions and has a valley width of ∿60 km.

Because of the method of construction, profiles 1 - 3 suffer from the limitation that wide valleys are also deep valleys and so would favor any analysis method that made this assumption. Observed profiles, on the other hand, tend to be wide, shallow and fairly irregular in contrast to the smooth variations in profiles 1 - 3, and might be expected to be less amenable to accurate analysis.

Profiles 4 - 6 have therefore been constructed using Arecibo incoherent scatter observations.

| | | | |
|---|---|---|---|
| (4) | Arecibo | 26 July 1972 | 062501-063000 LT |
| (5) | Arecibo | 26 July 1972 | 073957-074456 LT |
| (6) | Arecibo | 11 April 1972 | 170629-171105 LT |

Since the main concern is with the valley, the density distributions in the E and F layers have been assigned convenient analytic forms that agree well with the observations. A parabolic layer, which matches the observations closely near the layer peak, replaces the observed E layers. The observed F layer is replaced by a linear in $\ell n(N)$ layer which closely matches the base of the observed F layer.

The valley distribution is taken to be linear in $\ell n(N)$ between observed data points except near the E-layer peak, where it is assumed that the peak is parabolic for densities $\geq 0.9\ N_{max}E$, and the scale height is 10 percent greater than that used for the lower half of the peak. (In retrospect, it would seem that a figure of 40 percent agrees better with observations.)

Several F-layer distributions (linear in $\ell n(N)$ for profiles 4 - 6) favor some analysis methods; the remaining distributions (Chapman for profiles 1 - 3) favor none.

Twelve sets of analyses were requested, corresponding to two pairs of dip angle and gyrofrequency and to six profiles. The dip angle, gyrofrequency pairs were (20°, 1.2 MHz) and (70°, 1.5 MHz) for profiles 1 - 3 and (30°, 1.0 MHz) and (75°, 1.5 MHz) for profiles 4 - 6. The gyrofrequency was height-dependent, varying inversely as the cube of the height from the given value at 6371.2 km.

Some of the largest errors were obtained for profile 3, as illustrated in Figure 12. The results plotted in Figure 12 are representative of the general result that the methods of Paul and Titheridge are clearly the most accurate, the errors generally being less than a few kilometers and often much less. For the 20° dip cases, for example, the RMS errors immediately above the valley are 3.9 km (Paul) and 3.4 km (Titheridge). At foE + 0.3 MHz, these figures have dropped to 1.8 and 1.0 km, respectively. The error introduced by the use of the monotonic assumption would be equal to the valley width just above foE, decreasing rapidly towards foF2. If an assumed valley width is introduced, the error will depend on the agreement between the assumed and actual valley widths.

The Howe-McKinnis method used by Wright leads to underestimates of the height of the base of the F layer by about 10-20 km. The results indicated by "W" in Figures 10-15 were obtained using a constant gyrofrequency appropriate to an altitude of 200 km, the usual procedure followed with the use of this program, which does not allow for a height-dependent gyrofrequency. Wright has found that more accurate results can often be obtained by using instead the gyrofrequency at the height of the E-layer peak. These results are indicated by the dashed line in each of the figures. However, the errors obtained are usually still much greater than those obtained by Paul and Titheridge and do not seem to have arisen only from the use of a constant gyrofrequency.

The need for the height variation of the gyrofrequency to be taken into account in the analysis of the valley problem has not yet been investigated fully, but indications are that it is essential. Failure to do so results in the assignment of incorrect plasma frequencies to the X-ray frequencies used in the analysis.

Depending on the profile shape, Gulyaeva's method yields errors of 5-10 km, but in some cases overestimates of up to 30 km can occur. Both Gulyaeva and Wright intend modifying their methods in the light of experience gained in performing the tests.

## 4. SIMPLE METHODS OF ANALYSIS

Because of their proven accuracy, the methods of Titheridge or Paul should be used wherever possible. However, if suitable computing facilities are not available, recourse may be made to one of the simpler, but less accurate, methods of analysis, such as given by Schmerling [1967] and Titheridge [1967b, 1969].

As explained by Titheridge [1967b], his single polynomial method has certain practical advantages over Schmerling's [1967] 10-point method, as well as providing the extra information of the height of the maximum density, the scale height of the layer peak, and the sub-peak total electron content. The accuracy of the single polynomial method has accordingly been investigated by McNamara [1976], using accurate numerical ionograms (in fact the first set of numerical ionograms distributed to members in 1972-75).

For daytime monotonic profiles, it was found that (1) around foE the calculated heights were too low by about 10-15 km, (2) around foF2, the calculated heights were about 5-10 km too low, and (3) the error in the scale height was around $\pm 4$ km.

For nighttime ionograms, it was found that when allowance was made for underlying ionization the errors at the lowest heights were usually of the order of 10-20 km, depending on the dip angle and the value of foE.

For daytime non-monotonic profiles with an E-F valley, the errors yielded by the single polynomial method are of the order of the valley width, since the method assumes that the profile is monotonic. Similar errors would be expected for any simple method of analysis relying on the monotonic assumption.

The single polynomial method and the Schmerling 10-point method should be used only when suitable computer facilities are not available. If possible, attempts should be made to implement at least Titheridge's program, LAPOL, or Paul's earlier simplified method, both of which introduce simple underlying ionization and valley corrections and are smaller and faster than the more sophisticated methods. These methods are discussed further in Section 5.

## 5. CONCLUSION

For those members of the subgroup who completed all or most of the tests, the effort has been a learning process, the methods and computer programs being continually modified in order to perform better in following tests. In fact, neither of the most successful programs, Titheridge's POLAN and Paul's complicated method program, has yet reached its final stage. It is thus not possible to recommend a "best" method of N(h) analysis, although it is obvious that these two methods are the most promising.

If Paul's or Titheridge's method is used to analyze accurate nighttime ionograms with suitable X-ray traces, the height of the base of the F layer can generally be determined to within about 5-10 km. If either of these techniques is used to analyze an accurate daytime ionogram with suitable X-ray traces, the height of the F-layer base can be determined to within a few kilometers. Larger errors must be expected in the analysis of real ionograms, which introduce experimental errors. In fact, methods of N(h) analysis seem to have reached the stage where the accuracy of the results is limited by the quality of the input data, at least for analysis of conventional film ionograms.

For a method of N(h) analysis to be suitable for recommendation to users other than the originator, it must satisfy at least the following three conditions:

1. It should have been found reliable in all the tests conducted, these being assumed representative of typical ionospheric conditions.

2. The corresponding computer program should be

   a. readily available in standard FORTRAN,
   b. well documented,
   c. well commented throughout,
   d. easy to use,
   e. fast,
   f. as small as possible,
   g. provided with a complete set of test data and results.

3. The method should have well-defined and automatic default procedures for use when there is insufficient information in the ionogram (such as no suitable X traces for use in starting and valley corrections).

Since only Paul and Titheridge obtained good results for the valley tests, the Gulyaeva and Howe-McKinnis programs cannot be recommended for general use. However, the latter programs gave good results for the starting tests, and in some cases were more accurate than the Paul or Titheridge (POLAN) method. In fact, there were times when the Paul or Titheridge results were the worst, so both of these methods have their limitations. This point will not be discussed further because Paul did not perform all tests and Titheridge's POLAN has already been modified in the light of its relative difficulties with some of the tests.

At the time of writing, Titheridge's POLAN is more readily available and better documented than Paul's program. They require about the same computer storage and have very similar computation times. For some users, POLAN's "one-pass" solution may be an important consideration - at present Paul's program still requires visual inspection of the results to determine whether further iteration is necessary.

Both Paul's program and POLAN are fully supplied with default conditions, for use when there is insufficient information in the ionogram. This is an important consideration since these default conditions may be invoked in the majority of analyses. The relative merits of different default conditions is a topic for further study by the subgroup.

Computer programs and documentation will be available both from the originators and WDC-A in the near future. Some descriptions of POLAN are given by Titheridge in Appendix A.

For organizations with only small computer facilities, POLAN or Paul's complicated method (as used in the present tests) may be too large, in which case recourse must be made to programs such as Titheridge's simpler program LAPOL or Paul's earlier "simplified method for use with mini-computers." For these particular programs, accuracy of the results has been traded off in favor of simplicity, the starting and valley corrections being rather unsophisticated.

At the present time LAPOL requires slight modifications to prevent negative slopes at $f_{min}$, which arise occasionally when the calculated starting heights obtained using one X-ray point are too large. (This problem does not arise if the fixed frequency mode of LAPOL is used, as recommended by Titheridge.) Paul's simple method does not use X-ray data and is essentially the default version of his complicated method for use when no X-ray data are available. Both methods are available from their originators.

## 6. TOPICS FOR FURTHER STUDY

The following topics will be given the group's attention over the next 3 years:

1. Members will intercompare the default procedures they have adopted for the underlying ionization and valley problems. These methods are largely based on experience and are probably not capable of comparative testing, but a reasonable consensus view should be obtainable.

2. Members will make a determined attack on the problem of error bars, since the value of a result is greatly enhanced when its range of uncertainty is known.

3. A Handbook of N(h) Analysis will be prepared, illustrating with real ionograms typical problems encountered and their suggested solutions (with error bars).

4. Some discussion is necessary to determine whether it is feasible to develop a "best method of N(h) analysis," which would combine techniques used by different members. To this end, members will interchange programs and documentation.

5. A study will be made of methods for including angle of arrival information in the N(h) analysis, thus permitting allowance for horizontal gradients. Present methods of analysis assume that propagation is vertical, and this can cause errors, particularly in combined O-X solutions.

6. Procedures will be developed for including Z-trace information in the analysis of high-latitude ionograms.

7. Comparisons will be made of different methods of analysis of topside ionograms.

8. The "overlap" problem will be studied, using topside and bottomside ionograms, and TEC data.

9. A study will be made of problems of analysis which have been encountered at different dip angles.

10. Attempts will be made to develop an automatic (adaptive) method of analysis for sequences of ionograms.

11. A study will be made of the use of Es layers to provide information about the E-F valley.

Some of these topics are already under consideration by some members.

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Table 1. The real height errors (calculated value - correct value), in meters, obtained using 10 different methods of analysis

| FN | REAL HEIGHT | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.905 | 93.133 | 20 | -7 | 13 | -6 | 11 | 3 | -130 | 5 | 5 | -1 |
| 1.505 | 99.305 | -40 | 4 | 3 | 2 | 7 | 61 | 3 | 20 | 20 | 2 |
| 2.002 | 102.386 | -48 | 8 | 0 | 0 | 4 | 35 | 15 | 19 | 19 | 2 |
| 2.503 | 105.269 | -44 | 8 | -3 | -2 | 2 | -4 | -88 | 15 | 15 | 1 |
| 3.004 | 108.397 | -38 | 8 | -4 | -3 | 0 | -65 | -39 | 14 | 14 | 1 |
| 3.505 | 112.406 | -28 | 4 | -9 | -6 | -5 | -118 | -11 | 15 | 15 | 2 |
| 4.005 | 120.144 | 10 | 8 | -12 | -8 | -13 | -64 | 67 | 40 | 40 | 4 |
| 4.056 | 121.860 | 8 | 10 | -13 | -9 | -16 | 6 | 86 | 39 | 39 | 6 |
| 4.105 | 124.165 | -4 | 41 | 12 | 10 | -5 | 9 | 85 | 49 | 49 | 8 |
| 4.156 | 127.814 | 54 | 49 | 17 | 11 | 28 | -18 | 57 | 32 | 32 | 8 |
| 4.207 | 132.270 | 70 | -18 | -60 | -47 | -47 | -35 | 62 | 30 | 47 | 0 |
| 4.256 | 135.725 | 80 | 67 | 12 | 19 | -22 | -46 | 57 | 31 | 46 | 4 |
| 4.303 | 138.306 | 58 | 47 | -5 | -14 | 0 | -35 | 48 | 32 | 45 | 5 |
| 4.355 | 140.667 | 38 | 33 | 3 | 6 | -8 | 10 | 61 | 36 | 48 | 7 |
| 4.501 | 145.906 | 26 | 21 | 5 | 5 | -3 | 25 | 47 | 34 | 44 | 8 |
| 5.002 | 159.030 | -5 | 8 | -1 | -1 | -5 | -3 | 54 | 37 | 44 | 10 |
| 5.503 | 170.685 | -29 | 5 | -2 | -1 | -3 | -45 | 40 | 39 | 44 | 12 |
| 6.004 | 183.298 | -19 | 5 | -4 | -2 | -4 | -87 | 30 | 38 | 43 | 14 |
| 6.505 | 199.363 | 14 | 4 | -7 | -5 | -7 | -116 | 25 | 48 | 52 | 20 |
| 6.606 | 203.545 | 10 | 6 | -10 | -7 | -9 | -129 | 24 | 49 | 53 | 22 |
| 6.707 | 208.358 | 21 | 6 | -13 | -9 | -13 | -105 | 25 | 54 | 58 | 25 |
| 6.808 | 214.194 | 61 | 15 | -10 | -10 | -11 | -74 | 26 | 71 | 74 | 31 |
| 6.909 | 222.116 | 22 | -21 | -15 | -7 | 30 | -23 | 25 | 82 | 86 | 76 |
| 6.985 | 232.938 | 57 | 35 | 35 | 30 | 66 | -338 | 37 | 109 | 113 | -23 |
| RMS Errors | | 40 | 25 | 17 | 14 | 20 | 92 | 56 | 45 | 50 | 20 |
| No. of Points | | 51 | 51 | 49 | 49 | 49 | 38 | 85 | 56 | 56 | 142 |
| Min. Delta F | | .05 | .05 | .05 | .05 | .05 | .04 | .02 | .02 | .02 | .02 |
| Method | | I | I | I | I | I | F | F | F | F | F |
| Maximum Freq. | | 6.98 | 6.98 | 6.98 | 6.98 | 6.98 | 6.95 | 7.00 | 6.98 | 6.98 | 6.81 |

NOTE: The first two columns give the corresponding plasma frequencies and real heights; the columns marked 1-5 are Titheridge's results for LINAN, LAPOL, and POLAN, modes 3, 4, and 5; the remaining columns are the results due to Paul, Bossy, Gulyaeva (2 sets) and Becker. Also given for each method are the number of data points, the minimum frequency interval used near foE, the method of obtaining real heights at frequencies between the data points (see text), and the frequency above which the profile is fitted by an extrapolated peak.

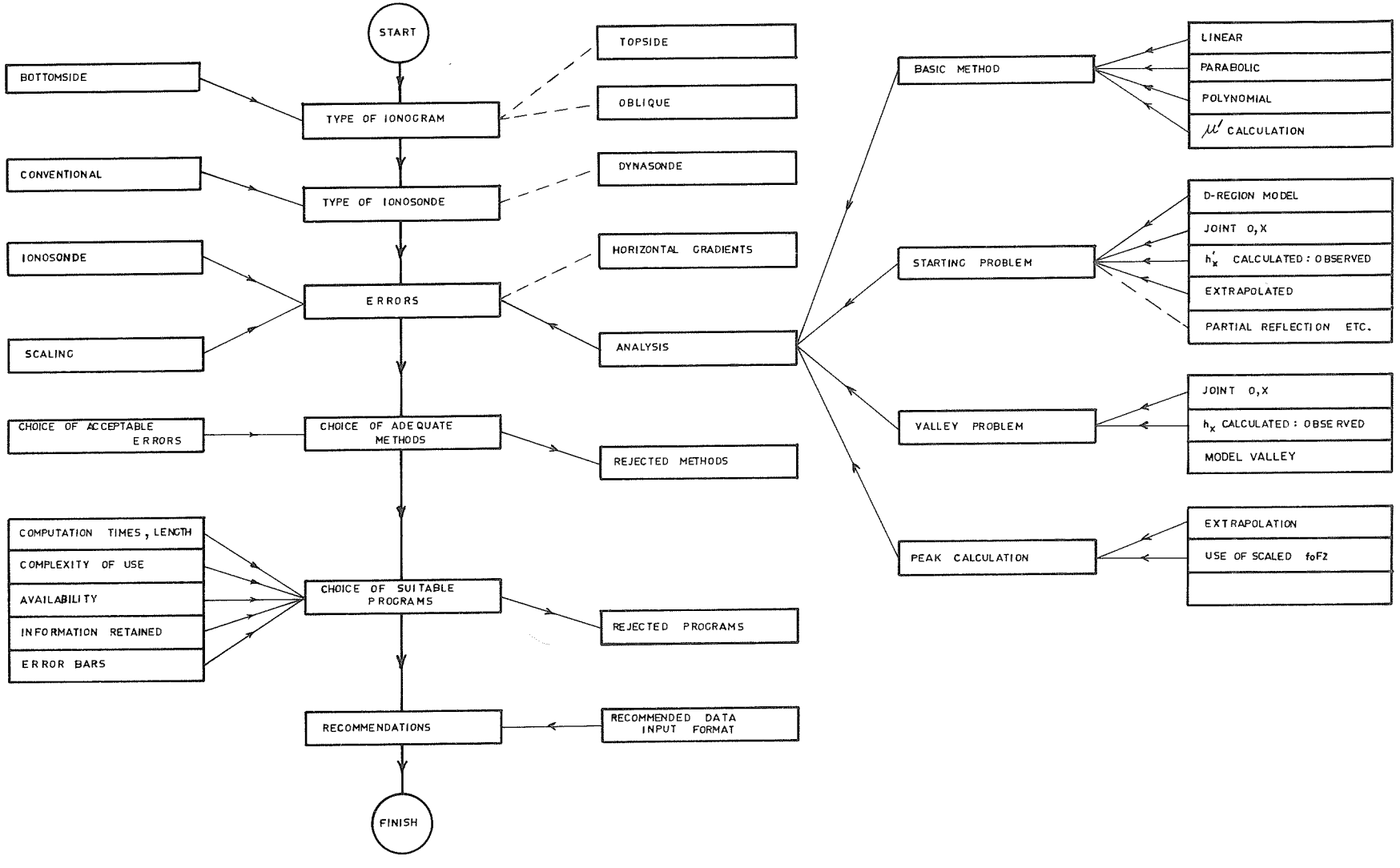


Fig. 1. General overview of the aspects of N(h) analysis of ionograms

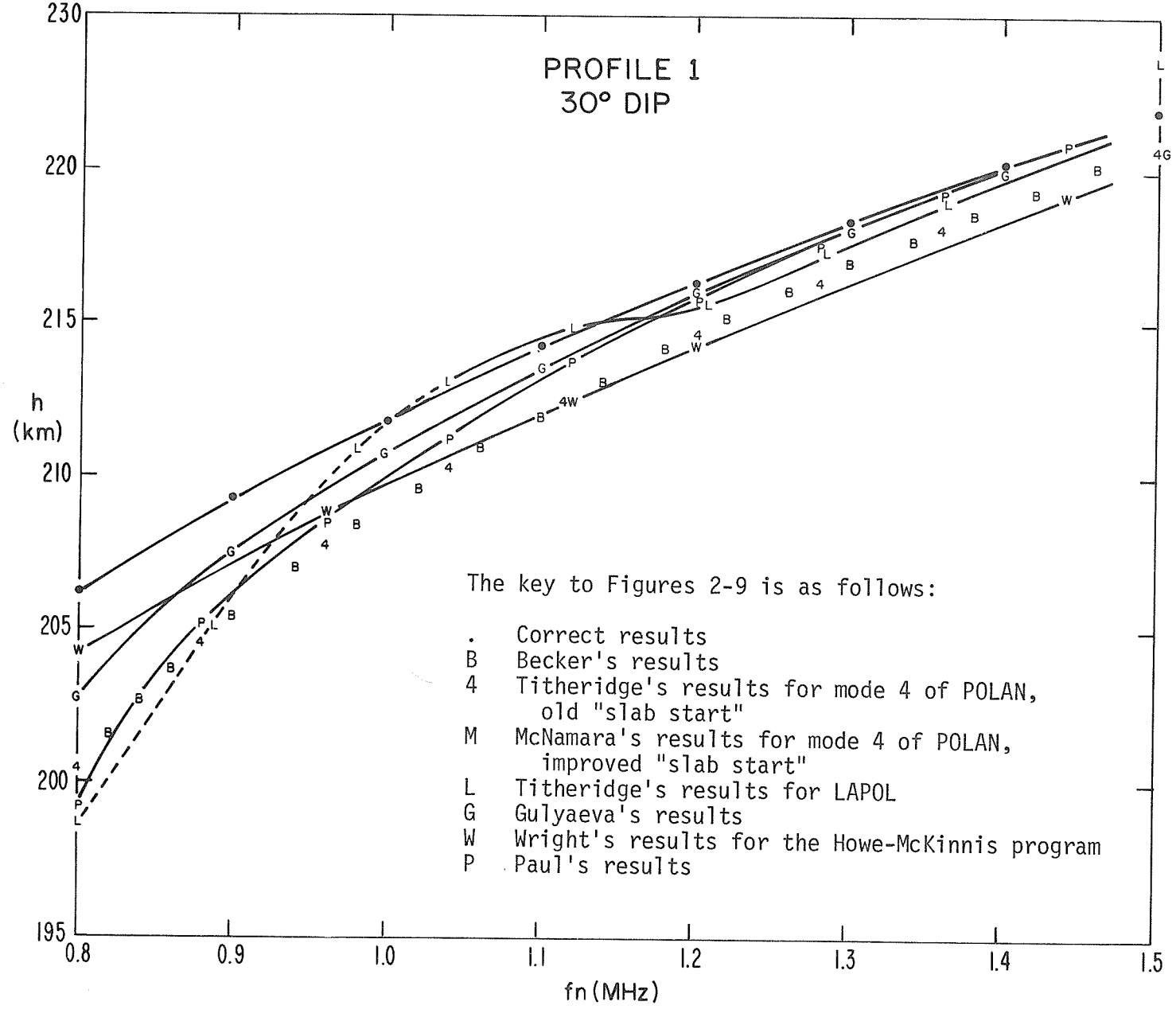


Fig. 2. Calculated and correct real heights for the lower parts of profile 1 used for the underlying ionization tests

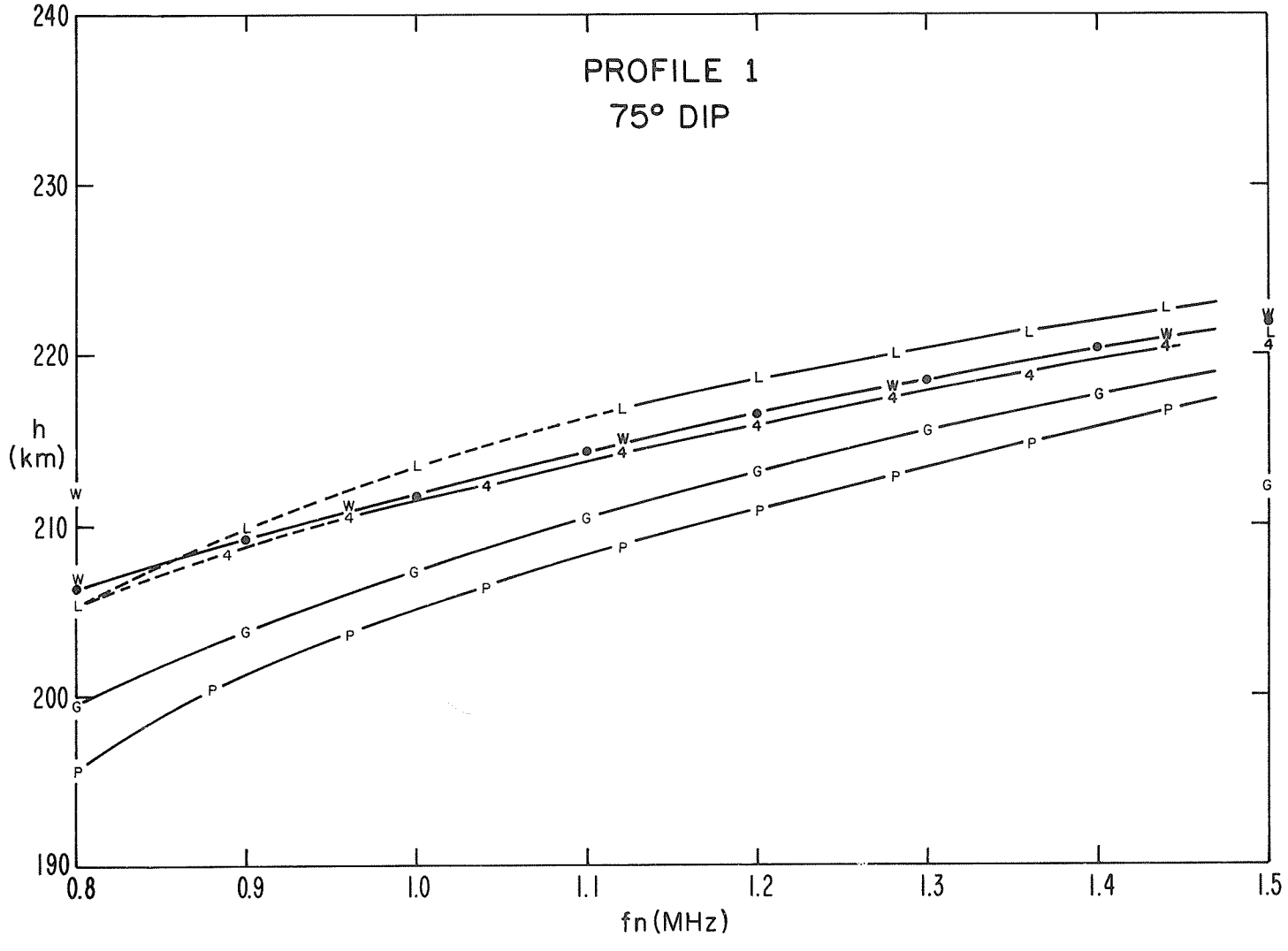


Fig. 3. Calculated and correct real heights for the lower parts of profile 1 used for the underlying ionization tests

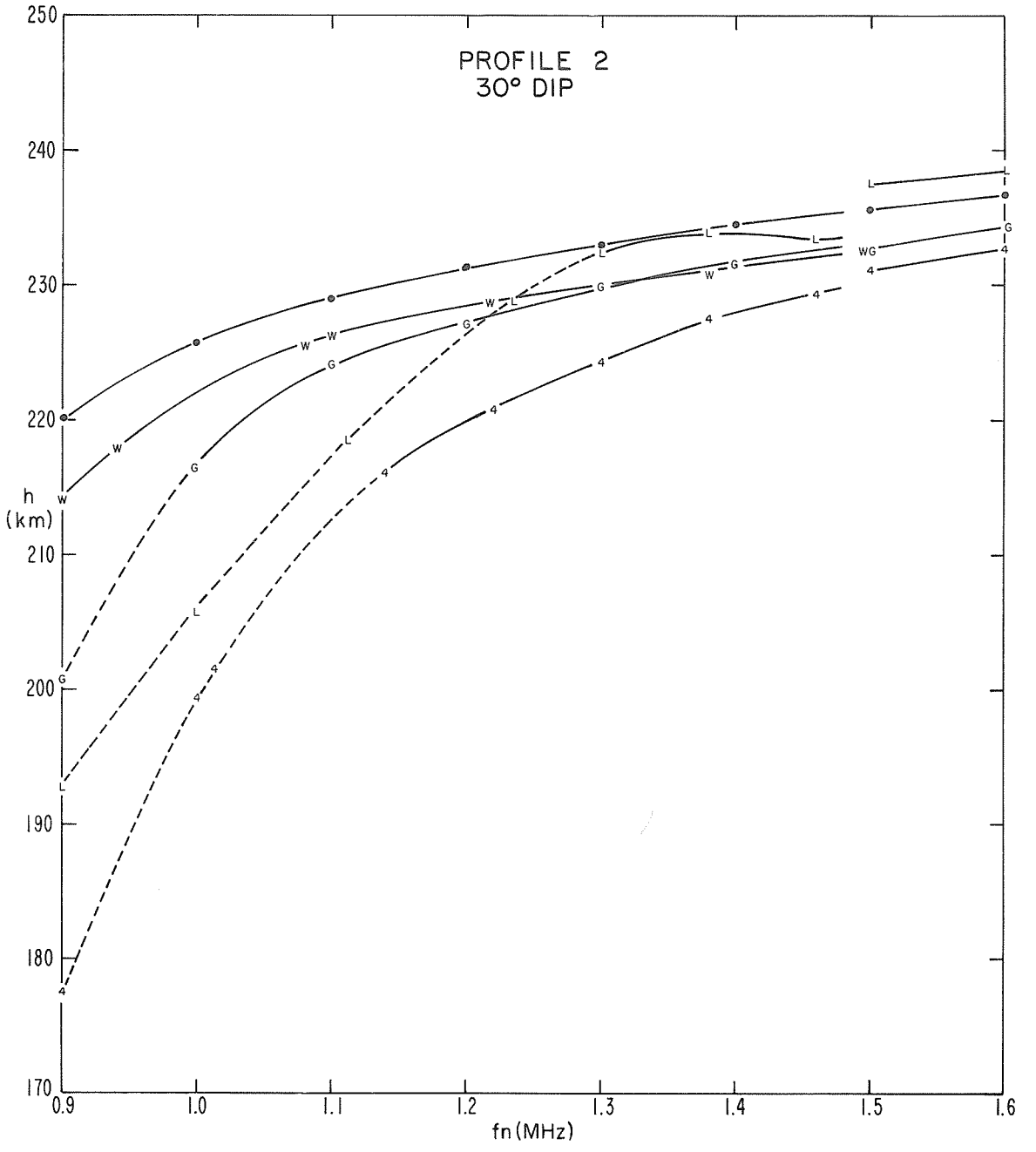


Fig. 4. Calculated and correct real heights for the lower parts of profile 2 used for the underlying ionization tests

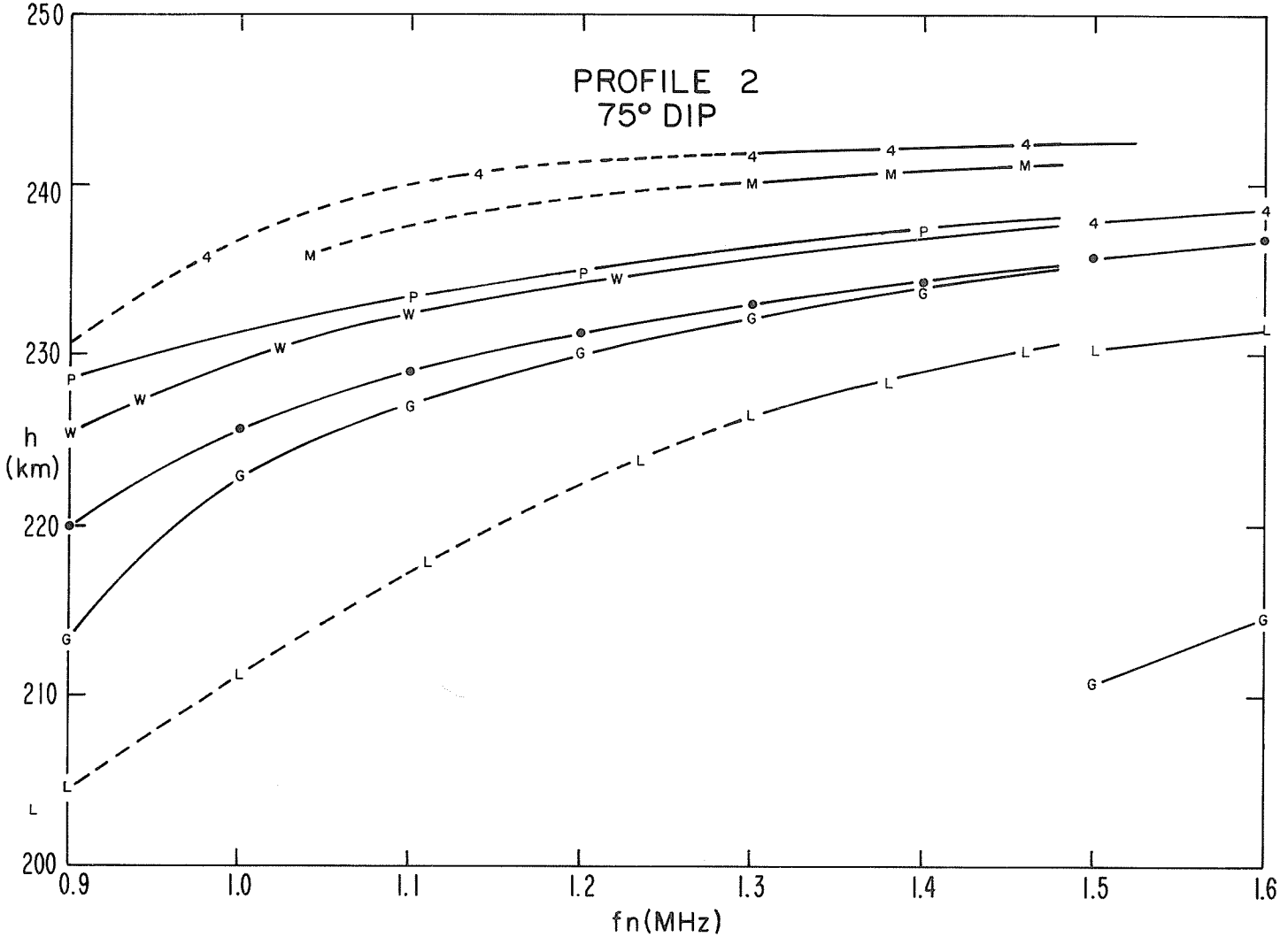


Fig. 5. Calculated and correct real heights for the lower parts of profile 2 used for the underlying ionization tests

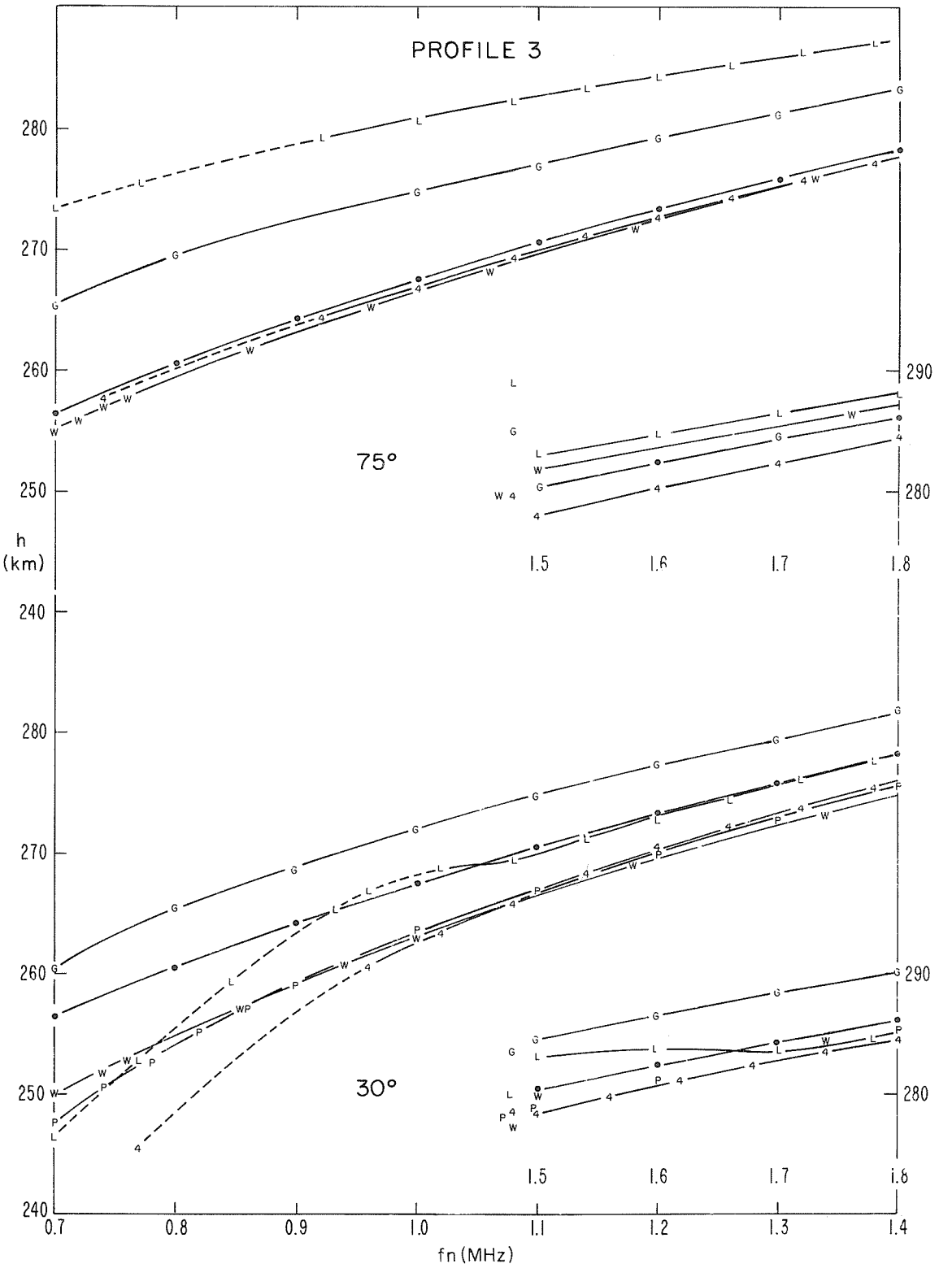


Fig. 6. Calculated and correct real heights for the lower parts of profile 3 used for the underlying ionization tests

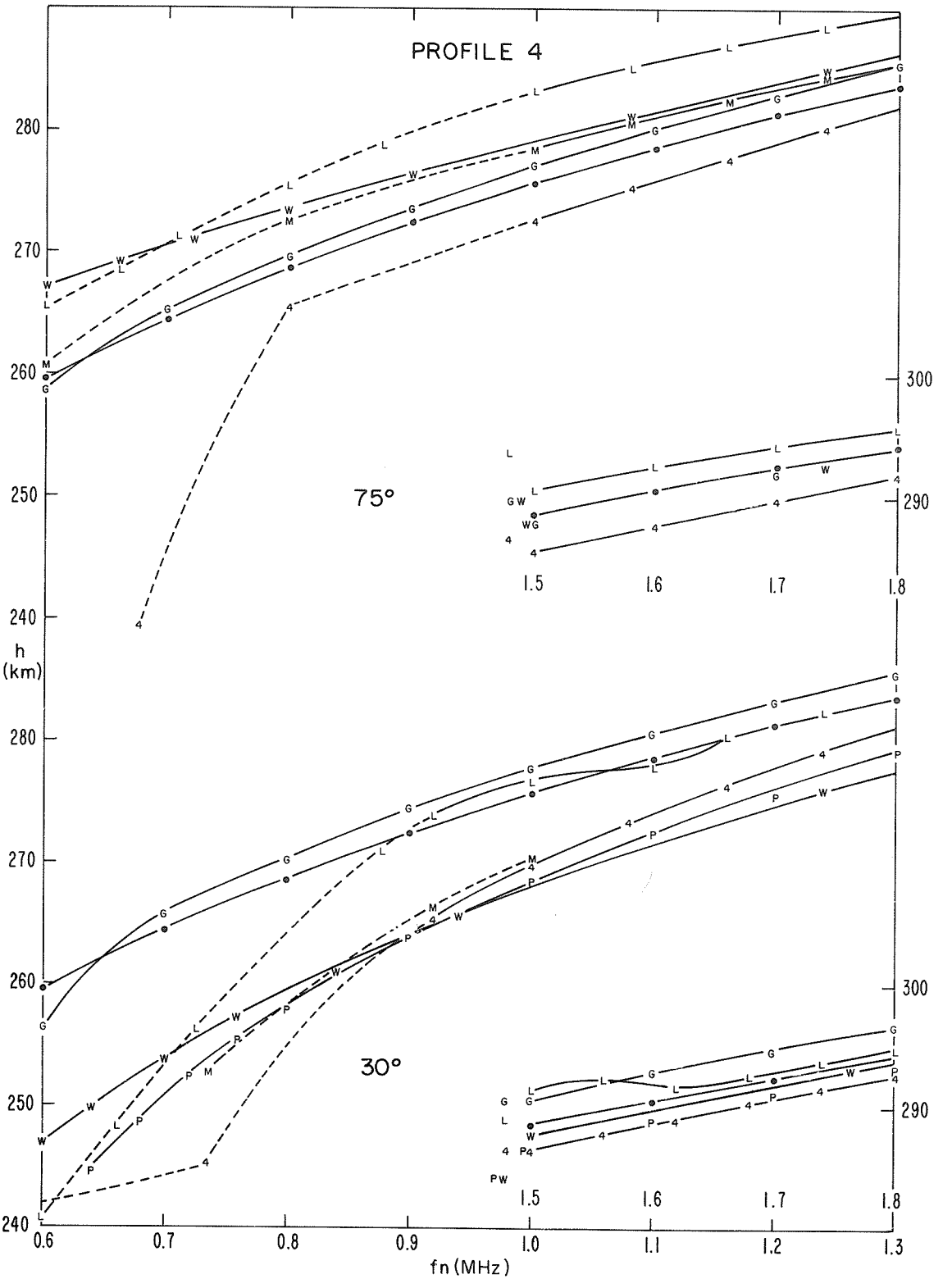


Fig. 7. Calculated and correct real heights for the lower parts of profile 4 used for the underlying ionization tests

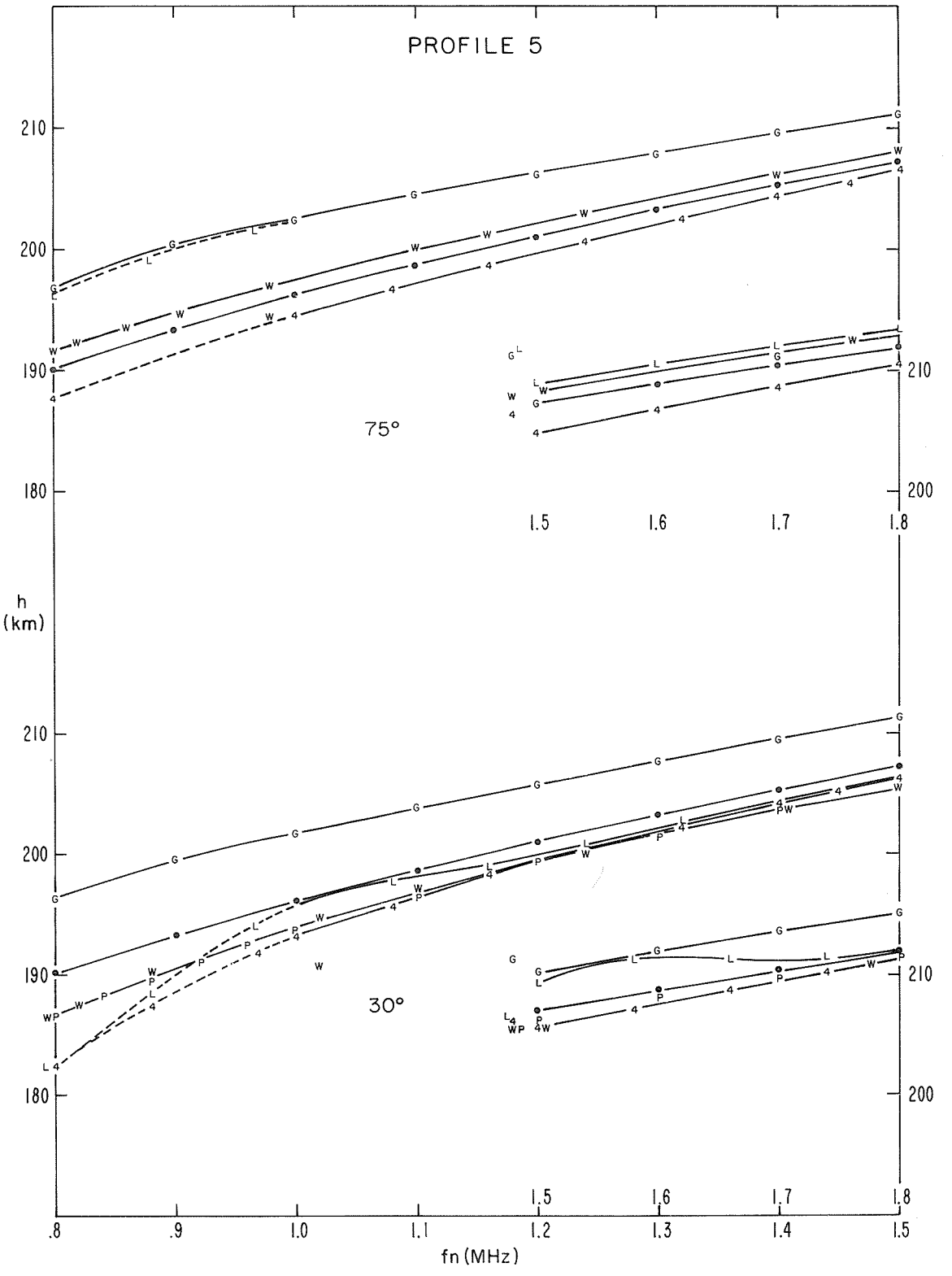


Fig. 8. Calculated and correct real heights for the lower parts of profile 5 used for the underlying ionization tests

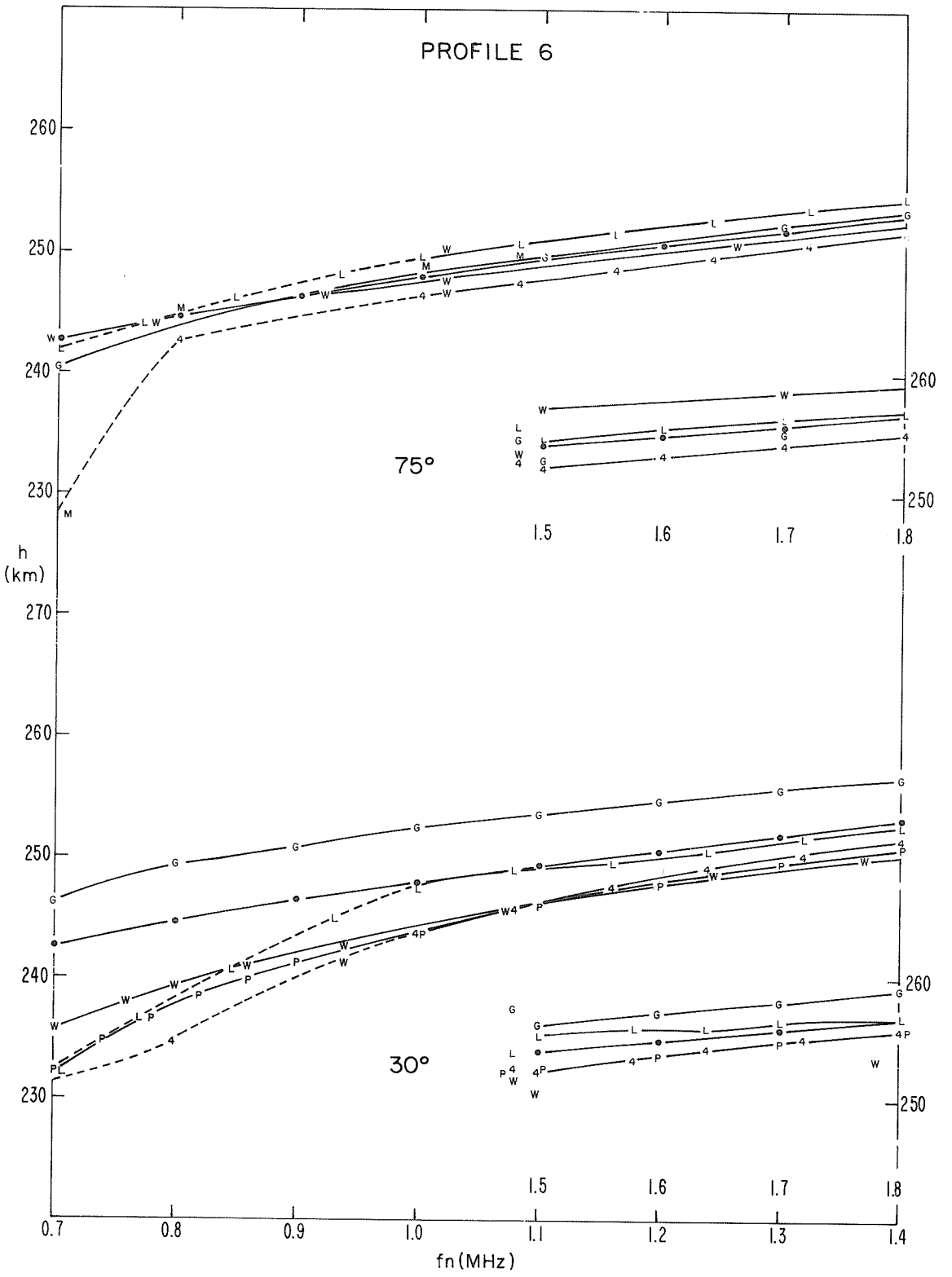


Fig. 9. Calculated and correct real heights for the lower parts of profile 6 used for the underlying ionization tests

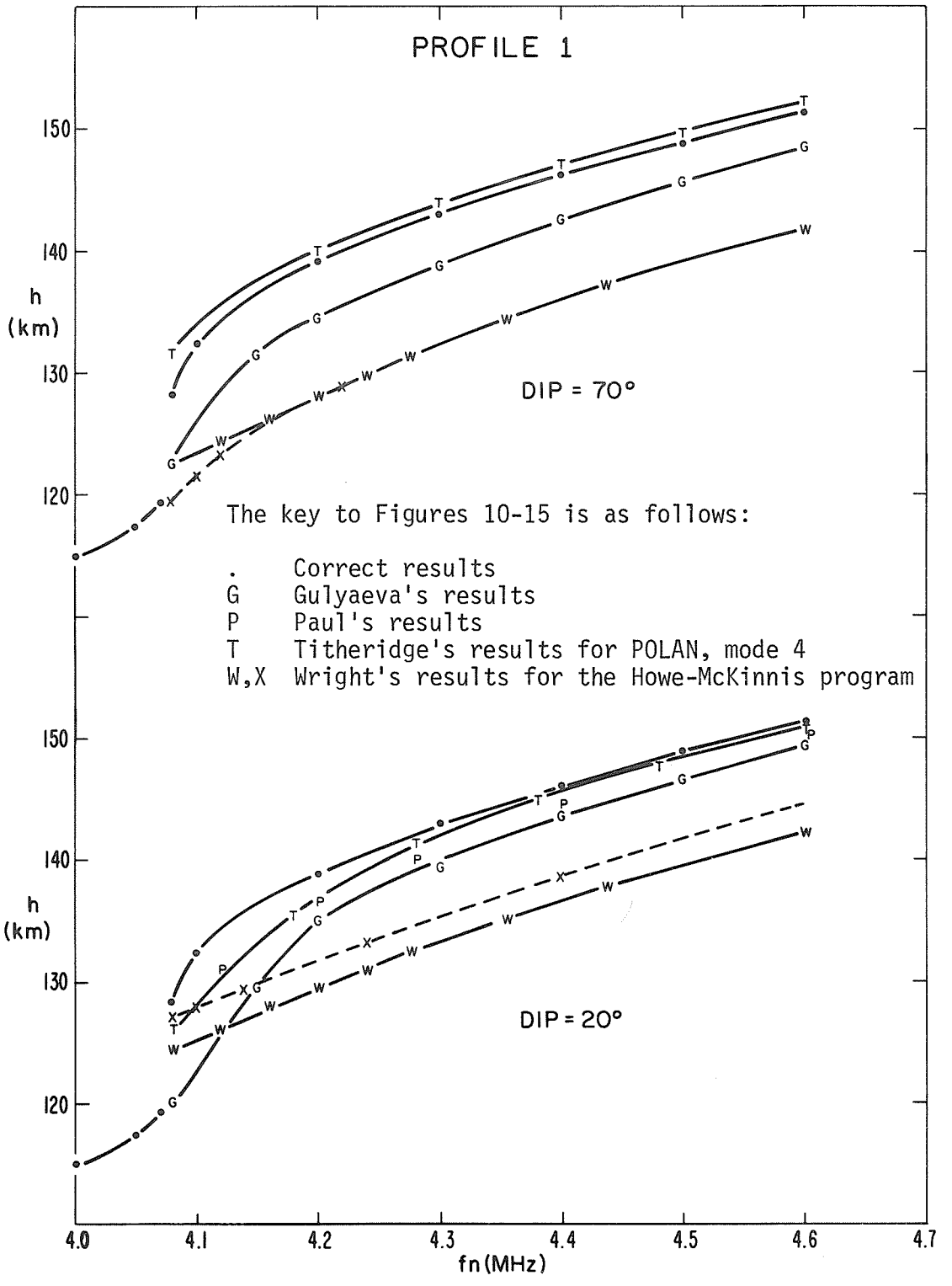


Fig. 10. Calculated and correct real heights for the base of the F layer for the profile 1 used for the valley tests

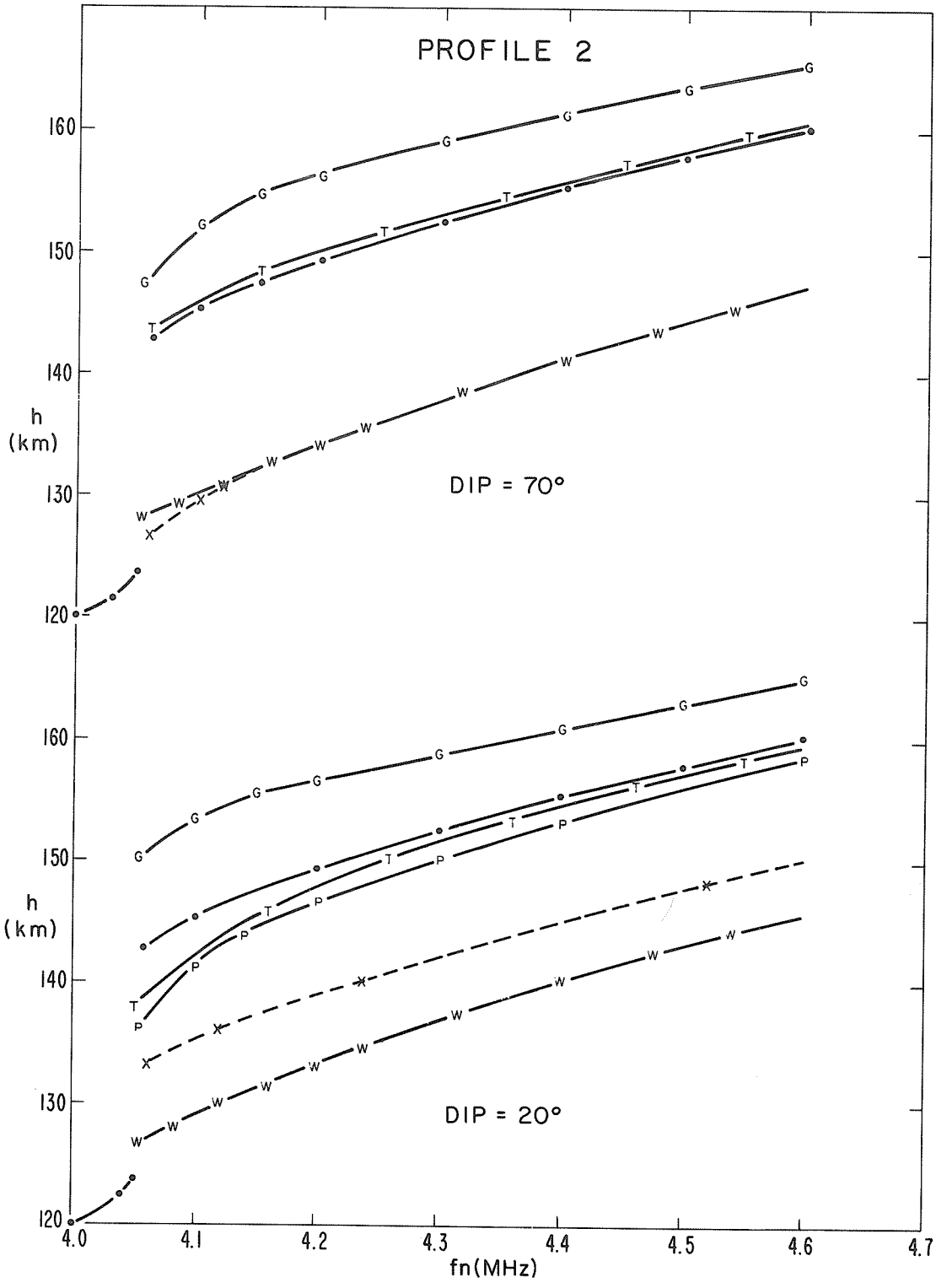


Fig. 11. Calculated and correct real heights for the base of the F layer for the profile 2 used for the valley tests

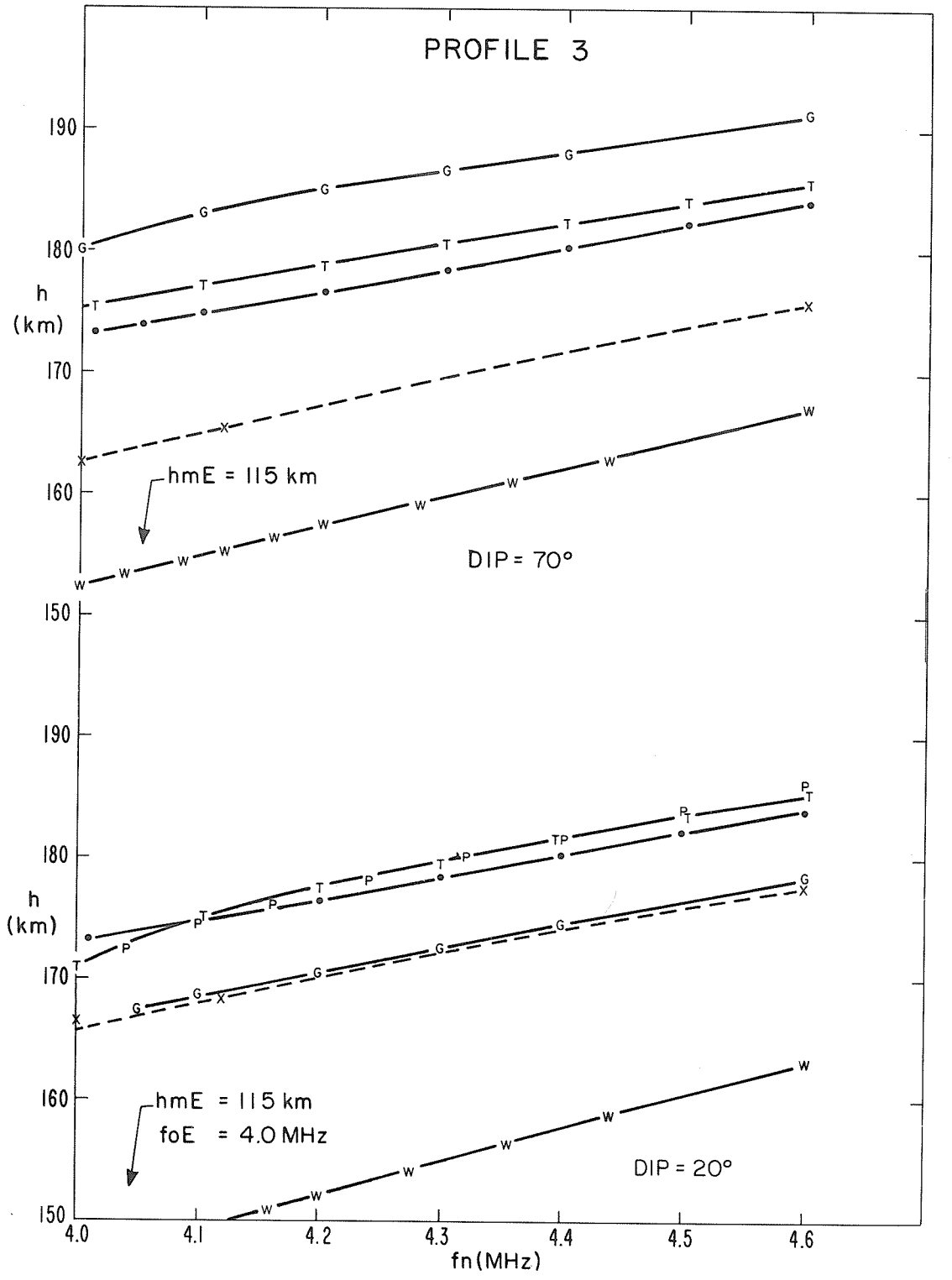


Fig. 12. Calculated and correct real heights for the base of the F layer for the profile 3 used for the valley tests.

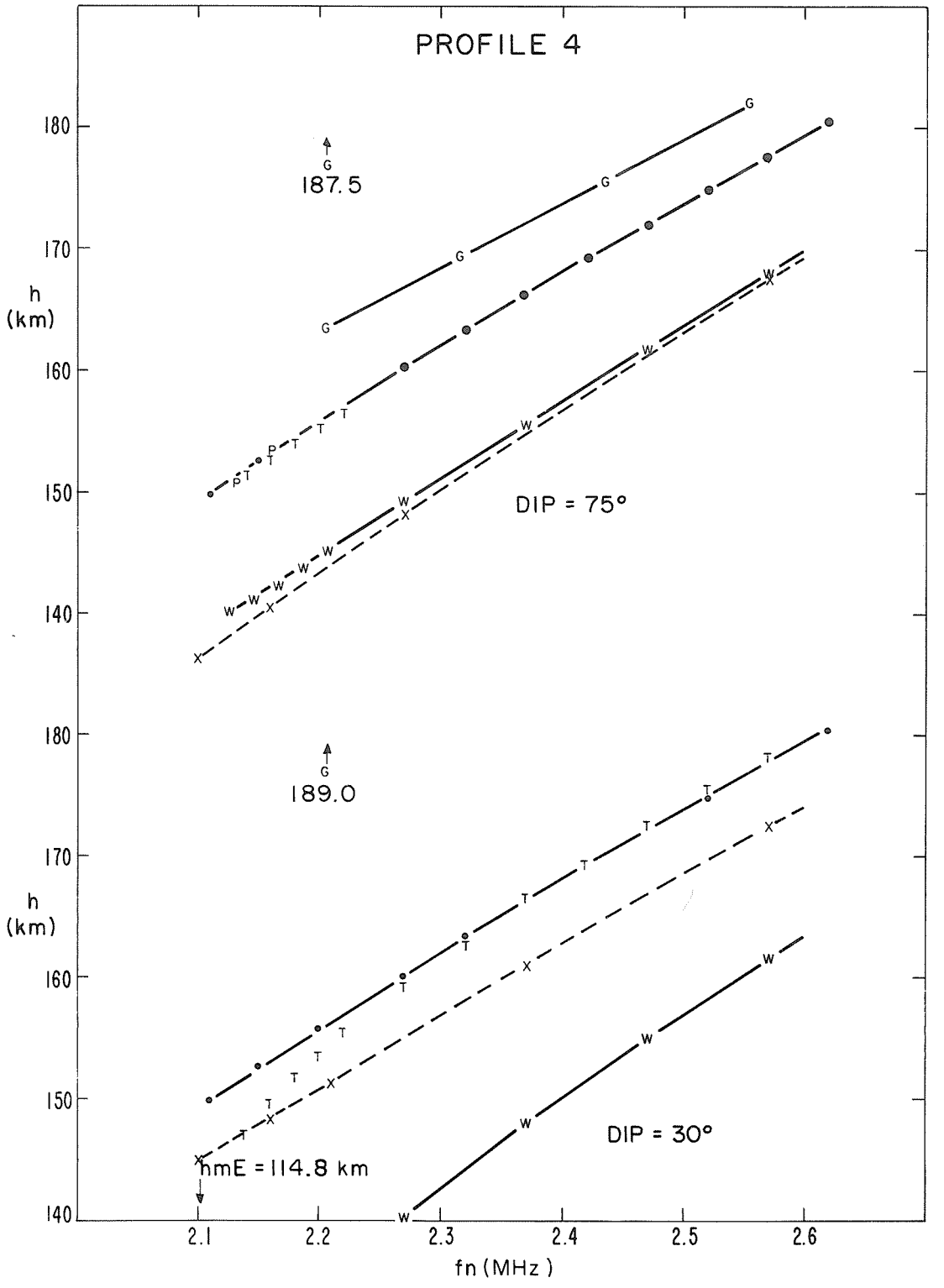


Fig. 13. Calculated and correct real heights for the base of the F layer for the profile 4 used for the valley tests.

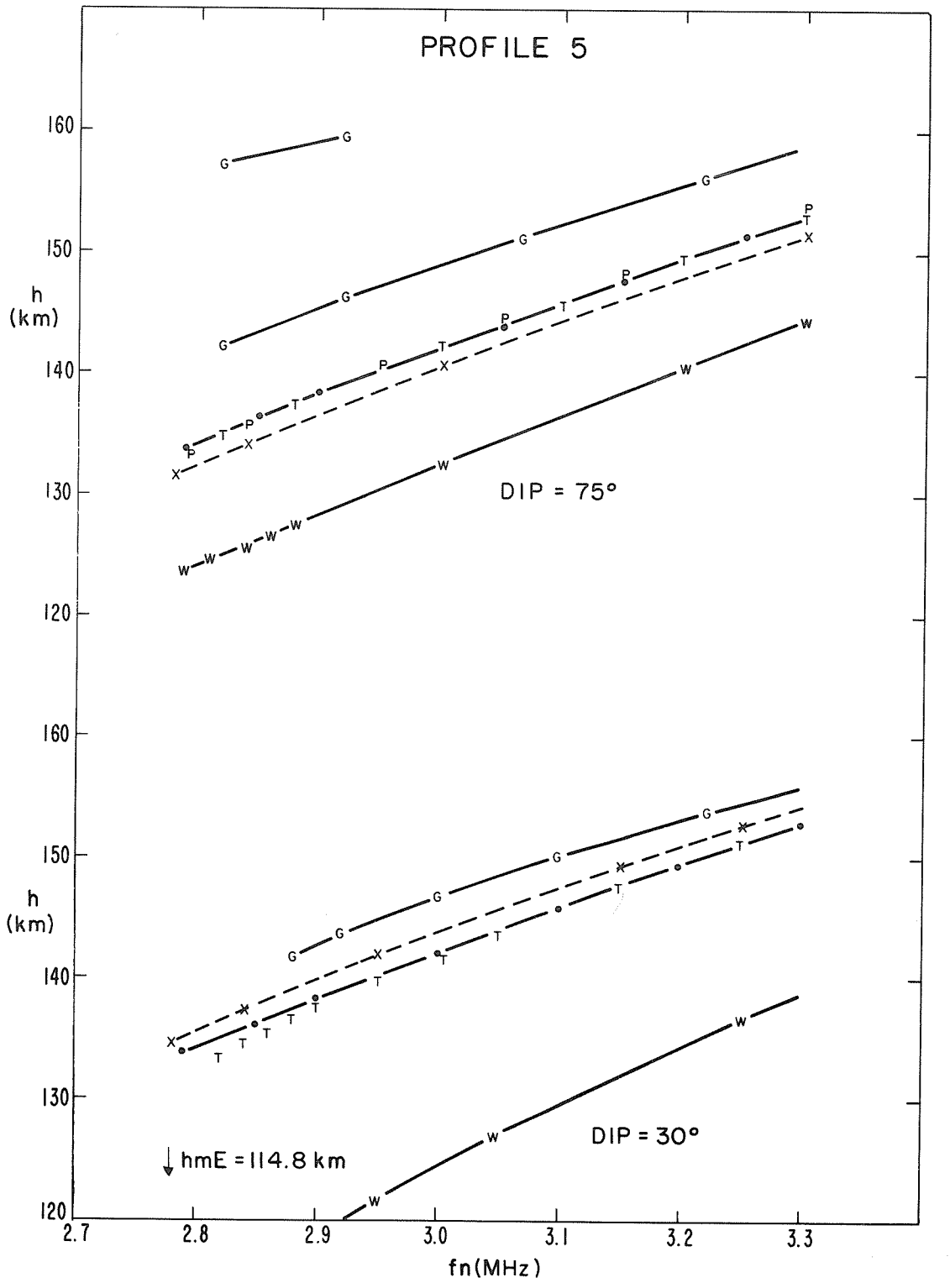


Fig. 14. Calculated and correct real heights for the base of the F layer for the profile 5 used for the valley tests.

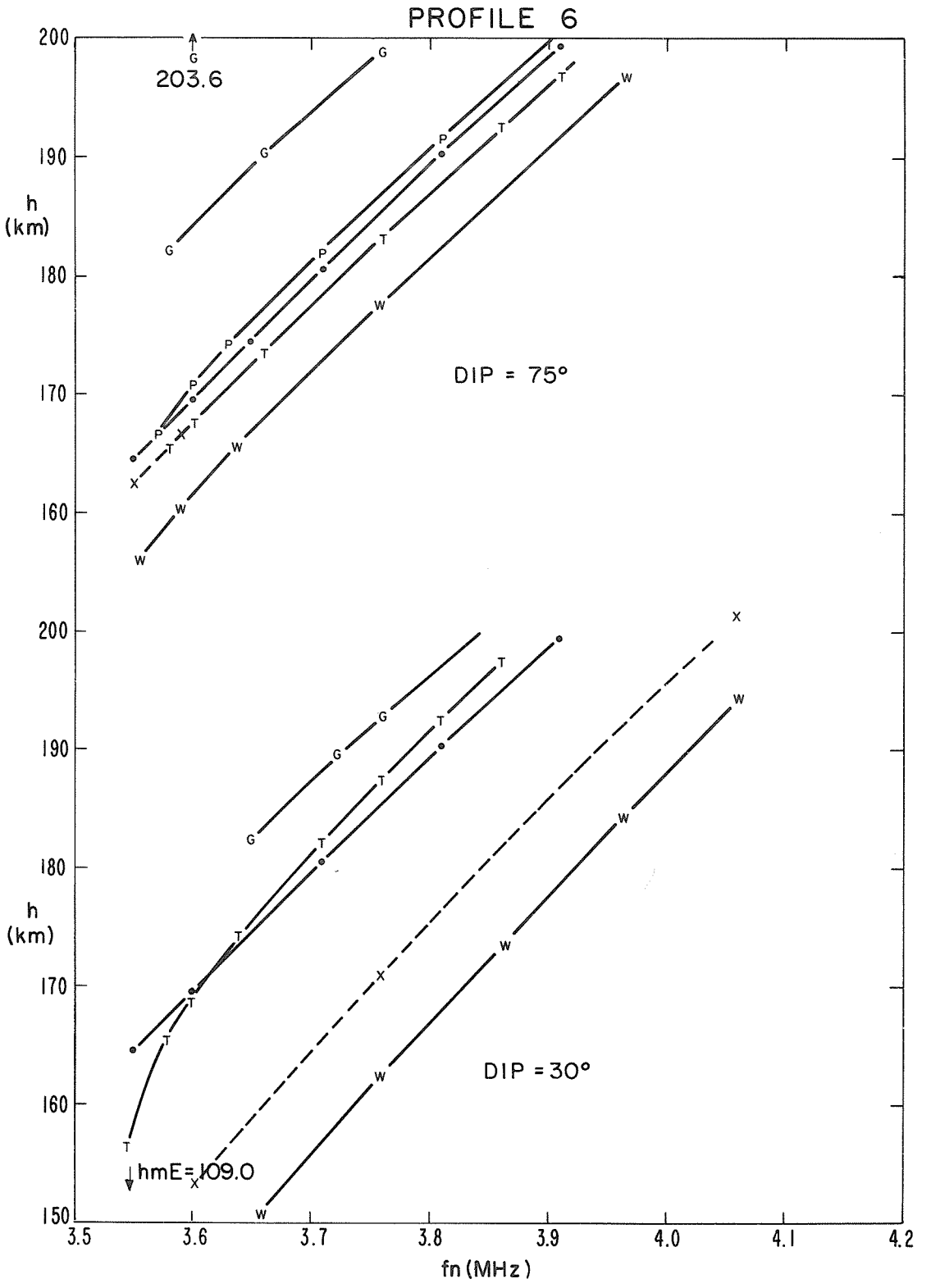


Fig. 15. Calculated and correct real heights for the base of the F layer for the profile 6 used for the valley tests.

# APPENDIX A

N(h) ANALYSIS by J.E. Titheridge - Historical survey and summary of results.

## 1. LINEAR LAMINATION METHODS

In the 1950's conversion of ionospheric h'(f) records to obtain the corresponding N(h) profiles employed the linear lamination approach. Analysis was done by hand using integrated group index coefficients interpolated from graphs or tables; consequently the simplest analysis took over 2 hours. It was shown (Titheridge, 1959b) that by calculating the changes in height from one frequency to the next a greatly simplified table of coefficients could be used. This speeded up the analysis by a factor of at least 10. The use of laminations in which h varied linearly with $1/f_N$ eliminated the need for calculation and subsequent integration of the group refractive index in deriving the necessary coefficients.

Using a different approach - replacing individual virtual heights by the average over all lower frequencies - the entire N(h) inversion problem can be formulated in terms of the phase refractive index $\mu$ (Titheridge, 1963). This appreciably speeds the analysis with, in general, a slight increase in accuracy. The relation between the phase and group paths, on which this analysis depends, is valid for any type of raypath (Titheridge, 1965).

The assumption of linear laminations produces large errors in the calculated real heights in regions where the gradient is changing rapidly (as at the peak). A simple rule was derived for determining, from the curvature of the calculated profile, the approximate size of the error at each point (Titheridge, 1961b, Appendix 1) so that a second order correction could be applied. A more direct method of countering errors in the lamination procedure is to calculate real heights at frequencies between the given data points. Thus an observed virtual height, which gives information primarily on the electron density gradient, is used to define the slope of a linear segment which extends either side of the observing frequency (Titheridge, 1978a). This simple modification to the normal linear lamination procedure gives in effect a second order analysis, with an accuracy as good or better than current (parabolic) methods.

## 2. HIGHER ORDER (POLYNOMIAL) METHODS

Further increases in the efficiency of N(h) analysis, to give greater accuracy from a given amount of data, require abandoning the 'linear lamination' assumption in which only the real heights are continuous across lamination boundaries. The logical next step is to use quadratic laminations, maintaining a continuity of gradient at all points. Would this, however, be the final best approach or might it in turn be replaced later by an even more sophisticated model? Thought was given as to what might be the end of such a process of successive refinement (Titheridge, 1959e). The ultimate model, at least for single-layer calculations, would seem to be one which maintained the continuity of all derivatives at all points. This implies the use of a single mathematical expression (e.g. a polynomial) to represent the entire profile. The mathematics implicit in this idea are tractable provided that the adopted expression is differentiable. For any set of scaling frequencies a matrix of coefficients can then be obtained giving the real height at each frequency directly in terms of the observed

virtual heights.

This procedure was used initially with a simple polynomial expression for the real height profile, the number of terms in the polynomial (i.e. the number of unknowns) being equal to the number of (h',f) data points (Titheridge, 1961b). Considerable improvement was later obtained by including a term which gives a parabolic peak at the observed critical frequency. The polynomial is constrained to become horizontal at this frequency, so that the peak is accurately parabolic (Titheridge, 1967b).

With the single-polynomial approach, representing the entire real height profile by a single analytic expression, coefficients can be determined which give any required parameters of the real height profile directly. Thus the height HM at the critical frequency, the scale height at the peak and the sub-peak electron content can be obtained directly from the measured virtual heights without the need for calculating any other aspects of the profile. For single-layer ionograms the results are of quite acceptable accuracy using only a small number of data points. Thus with a 5-point analysis calculated values of HM are an order of magnitude more accurate than those obtained using Kelso-Schmerling coefficients (Titheridge, 1966).

Improved stability in the single-polynomial analysis is obtained by assuming that at the lowest measured frequency the real height is equal to the virtual height. Coefficients have been calculated for the analysis of ionograms taken anywhere in the world, using 5 or 6 measured virtual heights (Titheridge, 1969). With this number of points the results are completely stable, and by choosing either the 5 or the 6 point frequency grid large cusps on the ionogram can be avoided. A third set of coefficients is provided for the analysis of night-time ionograms, in which the real height expression is fitted to 5 ordinary and 1 extraordinary ray measurement; the resulting profile is then approximately corrected for the effects of E region ionisation.

## 3. THE USE OF OVERLAPPING POLYNOMIALS

The single-polynomial analysis can be usefully employed with up to 8 measured virtual heights. With more than 10 data points the results become unstable. The problem is basically one of interpolation between given data points. The Kelso method applies a Gaussian interpolation to the virtual heights. In all other methods interpolation is done in the real height domain, since the real height curve is considerably smoother. The linear lamination analysis uses a linear interpolation between given frequencies to derive the relation between real and virtual heights. As in any similar problem of fitting discrete data points, accuracy is improved by an increase in the order of the interpolating polynomial. A limit is reached, however, beyond which the results become unstable. There is therefore an optimum number of terms (n) for the polynomial. When the number of data points to be fitted is greater than n the interpolation is done by using a different polynomial for each interval between successive data points.

In many problems in numerical analysis interpolating polynomials with 4 to 6 terms are about optimum. To determine the optimum number for ionogram analysis, coefficients were calculated in 1959 for polynomials of up to 8 terms (published as Table 2 of Titheridge, 1975a). These show that oscillatory tendencies begin with 7 terms at large magnetic dip angles, and with 6 terms near the equator. 5 terms were therefore adopted for the

polynomial used to represent the real height curve between successive data points.

Successive polynomials should ideally be joined by equating real heights and derivatives at one or more points on either side of the interval considered. Matching end points only gives a profile in which all derivatives up to n-2 are continuous. This corresponds to a spline function representation of the real height curve. Tests with different procedures showed, however, that fitting polynomials over a wider range gives slightly better accuracy and stability.

If fitting is done symetrically, in terms of real heights only, then a 30-point analysis with 5-term polynomials would require inversion of a matrix dimensioned $150 \times 30$ . This is scarcely desirable. Gradient matching at frequencies above the central interval of each polynomial is therefore replaced by matching of virtual heights. There seems little if any disadvantage in this approach, which enables a simple step-by-step analysis (Titheridge, 1961b, 1967c, 1974a). In general a $5 \times 5$ matrix must be inverted at each step. This matrix can become ill-conditioned, giving problems on computers with limited word length. By shifting the origin at each step to the last calculated real height, however, we obtain well conditioned $4 \times 4$ matrices giving errors (in the matrix inversion) of less than 1 in $10^6$ on 16-bit machines.

In tests using a number of different real height profiles with various frequency intervals and dip angles, the 5-term overlapping-polynomial analysis gives results which are 100 to 1000 times more accurate than the use of linear laminations, and 3 to 50 times more accurate than using parabolic laminations (Titheridge, 1978c). The stability of the analysis, measured by the amplitude of spurious real height oscillations following a cusp or discontinuity in the virtual height curve, is 20% to 50% better than for the parabolic analysis; if a gradient discontinuity is permitted at the cusp frequency, the oscillations (at or between scaled frequencies) are less than half as large as for the parabolic analysis (Titheridge, 1978b).

The ability to interpolate a point of inflection between successive frequencies means that selection of reading frequencies is not critical, and a fixed grid can be used for scaling ionograms. This considerably speeds up the scaling and, since coefficients need be calculated only once (for a given station) calculation of the real height curve takes typically less than one second. Both fixed and variable frequency modes, with an optional one-point extraordinary-ray starting correction and insertion of a model E-F valley, are provided in the program LAPOL (Titheridge, 1974a).

## 4. LEAST SQUARES SOLUTIONS

Further improvement in real height calculations requires the incorporation of more data points, which are suitably smoothed to remove the jitter caused by random errors. Optimum smoothing is obtained with polynomial methods of analysis using a least-squares solution, in which the number of terms in the polynomial (n) is less than the number of fitted data points (m) . There is then no limit on the value of m (apart from computer storage), while the detail required in the profile is set independently by the value of n . As a result of the least-squares procedure, fitted polynomials are completely stable for values of n up to at least 16.

For single-layer ionograms, all available data can be fitted in a single step. This is particularly valuable for the analysis of topside ionograms where full use can be made of fragmentary ordinary and extraordinary ray traces. These are incorporated into a single analysis which interpolates smoothly across any unobserved regions, and can give any desired number of points on the real height profile (Titheridge and Lobb, 1977).

With daytime bottomside ionograms single-polynomial solutions are obtained first for the E region and then for the F region. By incorporating a model valley into the analytic expression for the upper section, and using all available ordinary and extraordinary ray data for the F region, the best-fitting valley width and F region real height profile are obtained directly (Lobb and Titheridge, 1977a).

## 5. A GENERALISED FORMULATION OF THE N(h) INVERSION PROBLEM

All published methods of N(h) analysis (apart from the Kelso procedure) can be included in the following scheme. Virtual height data in the form of a series of points $(h'_i, f_i)$ is to be converted into a corresponding series of real heights $(h_i, f_i)$. Suppose real heights have been calculated up to the point k (corresponding to $h_k, f_k$). The next real height section from k to k+1 is defined by an expression which is a linear function of n variables. This is normally an n-term polynomial. The final 1 or 2 terms may however be replaced by a parabolic-peak expression, by terms which represent unseen ionisation with $f_N < f_1$, or by a term which specifies the width of a model valley (as in 7 below). The n-term real height expression is fitted to the last r calculated real heights, to the gradient dh/dN at the last s calculated points, and to v successive virtual heights. The virtual heights will include the point $(h'_{k+1}, f_{k+1})$, and may also include further points at both higher and lower frequencies.

These conditions specify a set of $m = r+s+v$ simultaneous equations in n unknowns. In the generalised polynomial analysis program POLAN the parameters r,s,v and n can be specified independently. A subroutine COEFIC is called at each step to calculate the $m \times n$ array of coefficients. For the real height and gradient conditions the coefficients are simple functions of the relevant frequencies. The virtual height equations involve group index integrals, which are evaluated using 5 or 12 point Gaussian integration depending on the desired accuracy. At high latitudes both 5 and 12 point integrals are employed, over different sections of the integration range, in a way which reduces errors by a factor of 10 to 100 at dip angles of 78 to 88°. At a given frequency the n polynomial integrals are obtained from the same 5,12 or 17 calculated values of $\mu'$. Calculation time therefore depends primarily on the number of virtual heights (v) fitted at each step, and not on the number of terms in the fitted polynomial.

Solution of the m simultaneous equations is carried out by a subroutine SOLVE. This gives an exact solution if $n=m$ and a least-squares solution if $n < m$. The calculation uses Householder transformations which give accurate results up to large values of n. From the n calculated real height coefficients any required features of the profile (e.g. the real heights at any desired frequencies, the total electron content,

the width of the valley or the scale height of a peak fitting section) can be obtained.

When a large number of data points are available, there is no need to use a separate polynomial for each interval between scaled frequencies. Expressions fitted over any desired range (including r real and v virtual heights) can be used to calculate a further NH real heights. NH is an additional free parameter in POLAN. For all current lamination procedures NH=1 . With digital ionosondes or automatic scaling procedures, however, larger values may be employed with advantage. Thus if the amount of available data is increased by a factor of 3, polynomials may still be fitted over the same frequency range as before (which will now involve 3 times as many data points) with each successive step in the analysis calculating 3 new real heights.

The parameters r,v,n and NH are set by 9-term data arrays IR, IV, IT and IH at the beginning of POLAN. A gradient fit at the last known real height (s=1) is used with MODE 2 (parabolic laminations) only, since it appears in general to have little advantage over matching virtual heights. 9 standard modes of analysis are then available by specifying a single parameter MODE when calling POLAN with the height, frequency arrays to be analysed. A wide range of other types of analysis can of course be obtained by changing the constants in the data arrays. The current arrays give the modes of analysis listed below. Starting and valley calculation procedures are incorporated with all modes of analysis when model start/valley constants are specified, or when extraordinary ray data is provided (as discussed in following sections). A detailed description of POLAN is given in Titheridge (1978c). This includes a listing of the computer program (which is also available on paper tape or punched cards); calculations of the accuracy and stability of the different modes; derivation, test results and evaluation of different start, peak and valley calculations under a wide range of conditions; and simple procedures for the accurate calculation of group refractive index and group index integrals at latitudes up to 89°.

Standard modes of analysis in POLAN:

MODE=1 Linear laminations (with a parabolic peak at critical frequencies).
MODE=2 Parabolic laminations, matching gradients (as described by Paul).
MODE=3 The 5-term overlapping polynomial analysis, fitting 2 real and 3 virtual heights at each step (Titheridge, 1961b, 1967c).
MODE=4 A least-squares fit of 6 terms to 7 data points (3 real + 4 virtual).
MODE=5 A least-squares fit of 6 terms to 8 data points (3 real + 5 virtual).
MODE=6 7 terms fitting 3 real + 6 virtual heights, advancing 2 frequency intervals at each step.
MODE=7 8 terms fitting 4 real + 8 virtual heights, advancing 2 points.
MODE=8 9 terms fitting 4 real + 9 virtual heights, advancing 3 points.
MODE=9 A single polynomial, of n terms, fitting all available virtual heights (0 or X ray, giving a total of v data points) with $n = 0.73(v+2)$ . Current dimensions limit n to 16 and v to 30. This is normally adequate since the analysis splits automatically into separate sections (with parabolic peaks) for each ionospheric layer. If more than 30 points occur in one layer, two (or more) polynomials are used, overlapped 4 points.

## 6. THE STARTING PROBLEM

### 6.1 Using ordinary (O) rays only

The major problem in ionogram analysis is the presence of 'unseen' regions, at low heights (where the plasma frequency is less than the lowest observing frequency $f_1$) or in a valley between two layers. Assuming a reasonably continuous profile, a non-zero value of $dh'/df$ near $f_1$ indicates the presence of group retardation. Extrapolation of $|dh'/df|$ to lower frequencies is therefore employed in POLAN, in the absence of any other starting information. The extrapolation range is kept small to ensure that there is no over correction. This procedure is considered a last resort, since reasonable profiles with large amounts of underlying ionisation can in fact produce zero values of $dh'/df$ .

A preferred system is the provision of a starting height, at 0.5 MHz, based on past experience or synoptic models. When this is done POLAN uses the lower of the given height and the height extrapolated from the O ray data, since the latter has been designed to give a minimum correction.

### 6.2 Using extraordinary (X) ray data

With an unlimited number of exact measurements on both components, the distribution of unseen ionisation (or its monotonic equivalent) can in theory be fully determined. In practise, however, only a limited amount of information is obtainable. Thus when $f_N < \frac{1}{2}f$ , $\mu_0'$ and $\mu_x'$ increase almost linearly with the electron density $N$ . The retardations of the O and X components then depend only on the total electron content $T$ of the underlying ionisation. Coefficients have been calculated which give $T$ , and the real height correction $h_1'-h_1$ , directly in terms of observed O and X ray virtual heights (Titheridge, 1959c). At high latitudes results are accurate to about 20% even if the condition $f_N < \frac{1}{2}f$ , for most of the unseen ionisation, is not fulfilled.

At dip angles near 29° the relative retardation of the O and X rays is approximately independent of $f_N$ , at frequencies below a few MHz . It is then impossible to determine anything about the distribution of the unseen ionisation. Only a single parameter can be obtained. This is however sufficient to define the retardation of the observed rays, so that the real height $h$ at any frequency can be obtained accurately and directly from corresponding values of $h_0'$ and $h_x'$ (Titheridge, 1974b).

At higher latitudes a relation of the type $\mu' = 1+aN+bN^r$ gives group retardations accurate to 1% for all $f_N < f-0.2$ MHz . Measurements on both components at frequencies near $f_1$ therefore define the values of $\int Ndh$ (=T) and $\int N^r dh$ for the ionisation with $f_N < f_s$ (where $f_s = f_1-0.2$ MHz) , and the mean gradient $G$ in the region $f_s < f_N < f_1$ (Titheridge, 1959c). With typical records only $T$ and $G$ can be usefully determined. These parameters are sufficient to give accurate heights for the reflecting region, and provide useful physical information on changes in the night E region (Titheridge, 1959d).

In many situations only a single parameter is meaningful. This can be found from a single value of $h_x'$ . Using an X ray frequency $f_x$ corres-

ponding to the lowest 0 ray frequency $f_1$ , the value of $h'_x - h'_1$ indicates the amount of underlying ionisation. A simple correction procedure is therefore to analyse the 0 ray trace beginning from a height $h_s$ at a frequency $f_s$ of about 0.5 MHz . Taking $h_s = h'_1 - C(h'_x - h'_1)$ a value of $C$ can be found such that the resulting profile agrees exactly with the virtual height $h'_x$ . It also agrees exactly with all 0 ray virtual heights. A general expression for $C$ as a function of dip, gyrofrequency and $f_1$ is derived in Titheridge (1961a).

For distributions of underlying ionisation with $f_N < 0.7f_1$ over most of the height range, the precise value of $f_x$ is not important. When the gradient $dh/df_N$ is large near $f_1$ , however, the correction becomes inaccurate. The error can be reduced by offsetting the value of $f_x$ , by an amount $\Delta f_x$ say (Titheridge, 1967c, appendix 2). With appropriate choice of the two parameters $C$ and $\Delta f_x$ results can be made exact for any two model profiles. They will also be exact for any linear combination of these profiles. Choosing reasonable models with and without large gradients near $f_1$ , the calculations can therefore be made insensitive to such gradients. Values of $C$ and $\Delta f_x$ have been determined numerically on this basis for a range of dip angles (Titheridge, 1975b). Approximating expressions are also given for computer analysis, and are included in the program LAPOL.

For topside ionograms a somewhat similar single-point measurement can be used to obtain directly information relating to the initial (top) section of the observed profile. The virtual depth of the 0 ray is measured at the local X ray frequency. Multiplication by a suitable constant then gives, with no further effort, the logarithmic slope (i.e. the scale height) just below the satellite (Titheridge, 1976).

## 6.3 Least-squares calculations

The program POLAN uses two parameters to define the underlying ionisation. These represent basically the total amount of ionisation with $f_N < f_s$ and the mean gradient in the range $f_s$ to $f_1$ . The first calculated polynomial begins at $f_s$ and gives in effect some additional information about the unseen ionisation, by assuming continuity of all derivatives across $f_1$ . Best results are then obtained with a rather lower value of $f_s$ , equal to $0.6f_1$ . Below this point POLAN assumes a linear variation from $0.3f_1$ to $0.6f_1$ . The thickness of this linear slab, and the constants of the first polynomial, are determined by a one-step least-squares fit to the initial 0 and X ray data points.

Typically (in mode 4) 7 parameters are obtained using 5 0 rays and 5 X rays. The first 5 parameters are the normal polynomial terms. The last two represent a constant term A (giving the amount by which the starting point $h_s$ is lower than the conservative 0 ray ex trapolation described in 6.1), and the thickness $D$ of the underlying linear slab. The subroutine

SOLVE is designed so that these final calculated parameters can if necessary be reset to zero, and corresponding adjustments made directly to the other calculated parameters and to the fitting error. If the calculated value of D is negative then it is set equal to zero and a new result obtained, corresponding to a fit of 6 parameters to the 10 virtual heights. Similarly if A comes out negative it also is eliminated from the solution. If, even after these adjustments, the initial gradient of the polynomial should be negative this (first) parameter is set to zero, the weight given to the X ray data is reduced, and the equations resolved. Thus the number of parameters defining the underlying ionisation is reduced as required to ensure that, even with poor data, the final profile is always physically acceptable.

## 7. VALLEY CALCULATIONS

Because of the higher frequencies involved, and the resulting decrease in $\mu_0'-\mu_X'$ , even less can be determined about the valley region. The one parameter of importance is the width of the valley. With practical ionograms, different assumptions about the valley shape give calculated widths varying by about 15% (Titheridge, 1959c). All these results fit the O ray data exactly and fit the X ray data to well within experimental error. Exact model calculations using square and triangular valleys of different depths gave a similar result, with errors of up to 20% in the widths calculated assuming a fixed shape and depth (Titheridge, 1960). Errors in the calculated F region heights are a similar percentage of the uncorrected error. Thus only a one-parameter valley calculation is normally feasible, and this will reduce the errors in calculated F-Layer heights by a factor of about 5.

At dip angles near 29° the same considerations apply as for the starting problem. No information can be obtained about the valley shape, but the width (and F region heights) can be obtained directly from corresponding O and X ray measurements (Titheridge, 1974b).

Applying a least-squares analysis to ionograms calculated from realistic model profiles, at a dip angle of 60°, showed that the virtual height data could be satisfied to within normal experimental error over a considerable range of assumed valley depths (Lobb and Titheridge, 1977a). In particular, the value of foE must be known to better than 1% before meaningful estimates of valley depth can be obtained. Because of the increased effect of errors in foE , and of the assumed valley shape and depth, the reliability of the calculated widths is decreased if data points are used within about 0.1 MHz of foE .

In the computer program POLAN a model valley is normally inserted above the E layer. To provide a reasonably realistic model the valley consists of three parts: (a) a parabolic section extending from the peak down to a plasma frequency of foE-D , using a scale height 25% greater than that calculated for the E layer; (b) a constant plasma frequency for a height range 0.6V; and (c) a linear increase between plasma frequencies foE-D and foE , over a height range 0.4V .

The total width W of the valley is V plus the height of the parabolic section. In the absence of X ray data we assume W = 20km and D = 0.05 MHz. These constants can however be changed as required, or the valley can be omitted.

With X ray data above a layer peak, the width of the valley is calculated (along with the constants of the first polynomial) by a least-squares fit to O and X ray data. The shape of the valley is maintained approximately constant, by setting $D \propto W^2$ . This requires an iterative procedure. 3 cycles are sufficient in all cases since large changes in the assumed depth D have comparatively little effect on the calculated width W.

With accurate virtual height data the program can determine W and D independently. This requires an expanded iteration cycle, to find the value of D which gives the smallest RMS deviation in the fit to the given virtual heights. As in the starting calculation, physical constraints are applied at each stage to ensure positive values for the widths V and W, and the initial gradient of the polynomial. With accurate data this provides a good estimate of valley depth at most dip angles. Near 29° dip the depth D may be far too great, but this has little effect on the calculated width and the F region heights.

## 8. BASIC ERRORS IN THE CALCULATED PROFILES

Apart from the starting and valley problems, errors arise due to inaccuracies in the data and to possible limitations in the theory on which the inversion process is based. It has been shown that, for a single magneto-ionic component, data errors must have a similar effect on all methods of analysis (Titheridge, 1975a). Further calculations show that random errors $\Delta h'$ in the virtual height data produce real height errors of the order of $0.5(\Delta f)^{0.6}\Delta h'$ , where $\Delta f$ is the interval between scaled frequencies (Titheridge, 1978c).

There are several possible sources of error in the basic relation $h' = \int \mu' dh$ (Titheridge, 1959e). Even in the absence of collisions, the ray theory assumed becomes inapplicable near the peak of a layer; the errors are however appreciable only within about 0.01% of the critical frequency, and so are of little concern.

Electron collisions in the ionosphere reduce $\mu'$ in the region near reflection, and can considerably decrease the height $h'$ given by the above integral. Under these conditions, however, $\mu$ is not zero at any real height and ray theory fails. In the absence of a magnetic field, and replacing the true profile by a linear approximation near reflection, integration in the complex plane shows that collisions have zero effect on the true value of $h'$ (Titheridge, 1961c). Numerical calculations show that this result is generally applicable. An exact virtual height is obtained by integrating the complex value of $\mu'$ , including the collision terms, up to the complex height at which $\mu = 0$ . At all dip angles this gives, to within 0.1%, the same result as integrating the no-collision expression for $\mu'$ up to the classical reflection height (Titheridge, 1967a). The effect of collisions should therefore be ignored at all stages in the analysis.

A further assumption inherent in the expression for $h'$ , that of a vertically propagating ray, can cause considerable errors. With an inclined magnetic field the O and X rays are deviated horizontally in opposite directions by a distance of typically 20km (Titheridge, 1959a). Detailed comparisons of $h'_X$ and $h'_O$ to obtain information about the unseen regions will therefore become unreliable in the presence of horizontal gradients in the ionosphere.

The presence of large irregularities in the ionosphere can cause horizontal deviations of over 100km (Lobb and Titheridge, 1977b). The rays avoid regions of decreased density. Thus a decrease travelling over a station may pass unseen on the ionograms. A similar increase will appear perhaps 200km larger (horizontally) than its true dimensions. In either case, when there is an irregularity within a few hundred km of the station, ionograms do not give a reliable measure of even the overhead critical frequency. Analysis of topside ionograms, taken in the presence of wavelike disturbances, gives most nearly the profile of the undisturbed ionosphere.

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# APPENDIX B

## Summary of N(h) Analysis Approach

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This work has developed out of the other well-known methods for N(h) analysis of ionograms. Since the implementation of the linear lamination analysis /1/ for the O-ray data, the method used at IZMIRAN has been considerably improved. The first attempt at combining the O and X data for the valley correction /3/ turned out to be unsuccessful due to the assumption of a many-parameters model in the E-F region that produced ambiguous N(h) results in many cases. Fairly good criteria for conforming the N(h) results to the O and X echoes were established when a one-parameter model of starting and valley ionization was adopted /4,5,6,10,16/.

Because we intended to develop a program for the users (rather than for the originator), we prefer an iterative approach /6,8,16/ to the least-squares strategy when both O and X data are available, since the former gives less breakdowns in practice though at the cost of some additional computation times required for performing the iterations. The modification of the iterative strategy /16/ leads to large savings of computation time.

Since the Special Issue of Radio Science /2/, efforts have been continually made to test the analysis methods /4,6,9,14/, to compare N(h) profiles deduced from the ionograms with the data of rocket measurements /7,17/, and to unify the different methods of N(h) analysis /8,18/. While developing the method of combined use of O and X data, the urgent need was discovered for recommendations concerning parameters for the start and valley in the most common cases when only O-ray data were available. The conclusion was drawn that further progress in the field must be founded on study of the relationship of realistic N(h) profiles to the corresponding numerical ionograms, and several studies were carried out in this direction /11,12,13,15,18/. Gulyaeva /18/ suggests a procedure for starting and valley corrections using the observable parameters on each ionogram when only one magnetoionic component is available.

Progress of the methods for computing electron density profiles from the vertical-incidence ionograms permits us to lay down the key principles, which must be satisfied by a good program for N(h) analysis:

1) High accuracy of N(h) approximation over a frequency interval;

2) The best possible accuracy of allowance for the underlying and valley ionization;

3) Simplicity of the algorithm through a little number of unknown parameters of "unseen" ionization;

4) Full-automatic (free of a human intervention) search for the N(h) solution among the set of the possible ones using the rigorously-established criteria;

5) Minimum of information required in addition to the scaled h'(f) data points (the most attractive being a version when only geomagnetic parameters DIP and $f_H$ are required); and

6) Minimal computation times for obtaining each N(h) profile.

Also, the program must be sufficiently flexible to handle any number of layers (and valleys) in the ionograms, with either combination of O and/or X echoes in each layer. It is desirable to provide for the alternative when the given ionogram is accompanied by some given part of the real N(h) distribution obtained, for example, from the rocket experiments or incoherent scatter observations, or from ionospheric models. All obtainable ionospheric parameters along with the more obvious h(f) results should be provided in the computer output. The program must be written in one of the international algorithmic languages such as ALGOL or FORTRAN.

The first published ALGOL program /5/ and its modification for the second order method /10/ complied with many of these requirements. Continued employment of both programs in practice and comparison of those with other well-known programs for N(h) analysis /14/ indicated that the present method must and can be improved in some rather important areas.

To make it more comparable to and interchangeable with the other programs for N(h) analysis, it was decided to develop this improved version of the program in FORTRAN /16/. This work was supported by the management of WDC-B where the large archives of ionograms and FORTRAN-oriented computer are available. The program ITERAN has been tested with Tests 5-9 of URSI Subgroup G/6/2.

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# APPENDIX C

## Conversion Methods for h'(f) to N(h) Used at Lindau

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### 1 Methods Used Before 1967

Real-height analysis of multifrequency ordinary and extraordinary virtual-height traces has been carried out at Lindau since 1956 with methods of our own design. All of them take account of the influence of the earth's magnetic field, even the model method we started with in 1956. It merged into a more general manual method allowing the conversion of any monotonic ordinary or extraordinary h'(f)-trace. In 1961, an IBM-650 computer became available to us. Just as the manual methods had done, here also a height-independent geomagnetic field was assumed. Underlying ionization was taken into account in the same way as the manual methods had done. But the critical frequencies were no longer estimated by eye; they were analytically deduced. By doing so, a better accuracy of the layer peak parameters could be attained. The method itself was a laminar method with linear electron density increases within each layer step. This program was mainly intended to continue our studies on the solar control of the F-region and its dynamics throughout half a sunspot cycle.

All the methods just mentioned and the physical results deduced with them were reviewed by the author in 1967. Through an oversight, all the references of this paper had been put together with those of my former assistant and attached to his paper (Herbert, 1967).

### 2 Methods Used Since 1967

In 1964, an IBM-7040 computer replaced the IBM-650 machine and allowed us a further improvement of the accuracy of our results, to store them on magnetic tapes and to provide many other options. Concerning higher accuracy, the former linear electron density increases within each layer step were replaced by second order polynomial approximations with equal gradients at the common boundary of adjacent layer steps. Now the height dependence of the geomagnetic field could be taken into account also.

In 1969, we began to use an IBM-360/91 computer in addition. Thus, we could start extensive studies of the topside ionosphere by converting a large number of Alouette I and ISIS2 ionograms. For that purpose a new semi-automatic scaling facility had to be designed first. It is shown in Figure 1.

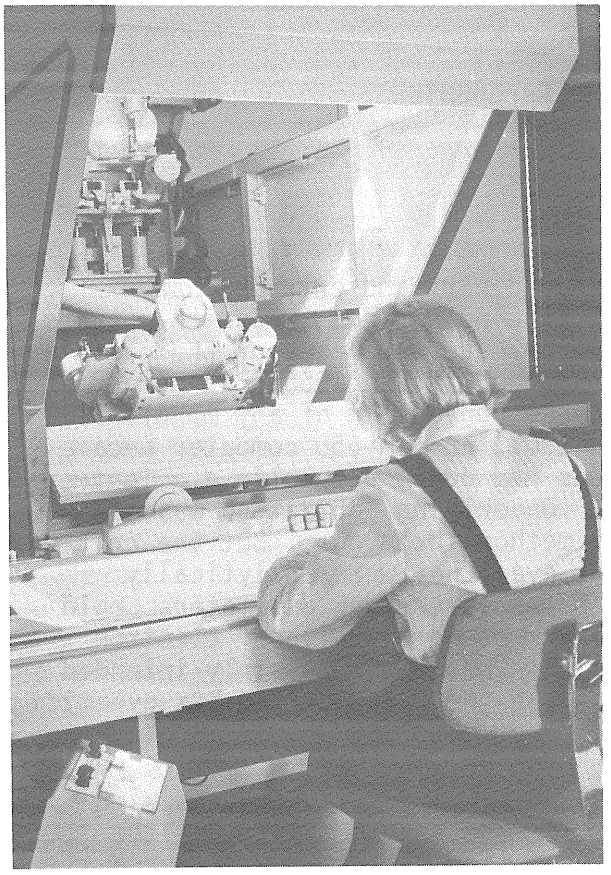

Fig. 1
Semi-automatic scaling facility

This machine allows us to digitize film records between 8 and 35 mm, with film images of any length and any coordinate systems. A versatile optical system (Zeiss, Oberkochen, Germany) projects the records onto the table of a D-MAC Pencil-Follower (Edinburgh, Scotland). The enlargement can be varied not only by replaceable optical systems but also by shifting the projector back and forth mechanically. In order to adjust the projected ionograms to calibrated markers on the table of the pencil-follower, the projector can also be shifted along the other two coordinate axes whereas the film is kept by two glass plates pressed together; they release the film as soon as the respective push button for film transport (low or high speed, unrolling or rewinding) is pressed. For data input such as name of the station, name of the satellite, date and time, etc., a second disk is installed. The mechanical system on the pencil-follower facilitates the digitization of records with non-rectangular virtual-height/frequency coordinate systems. The second projector can be used for review purposes, since very long ionograms must be scaled section by section. Several computer programs have been written for this data handling process. They allow the input data to be transformed into pairs of frequencies and virtual-heights, to check their formats and finally to get them corrected by the computer according to punch card instructions.

Also, the h'(f)/N(h)-conversion method had to be extended for this new project.

Now, our scaling personnel need only to punch the name of the satellite under consideration and the date and time of observation. The computer then determines the other data wanted, such as position of the satellite, the intensity and inclination of the earth's magnetic field at any height, etc. The IBM-360/91 computer allows us to make use of all available information given by the ordinary, extraordinary, and z traces. Our present program also allows us to use just sections of each trace, since for quality reasons a mode change might improve the quality of the results.

For timesaving and practical reasons, no restrictions have been made as far as the total number of readings per ionogram or the frequency spacing is concerned. The scaler is just urged to scale more data the steeper the gradient of the trace under consideration. Special data needed by the analysis are derived by interpolation without smoothing out any real peaks.

Beyond 800 km topside profiles are approximated by exponential layer steps and at lower heights by second order polynomial fittings, just as for bottomside profiles. The underlying ionization problem in topside analysis work has been solved in the same formal way as in bottomside studies. An exponential electron density/height dependence replaces the cosine layer step there.

The information given by Es-traces well above the peak height of the normal E-layer has also been taken into account. Thus, the semi-thickness of a parabolic approximation of the topside E-layer and the plasma frequency where the Es-layer emerges can be determined. A short review of that chapter of our h'(f) to N(h) conversion efforts was published by the author in 1969.

## 3 Methods Under Design

Our recent project deals with a detailed study of the valley problem and with full-day electron density/real-height profile variations. The necessary subroutines are now being submitted to their final tests.

In this project, we have been confronted with two key problems: the underlying ionization problem in its most general form and the valley problem. We have tried to solve them as shown in the next section.

### 3.1 Underlying Ionization Problem

Seven layer steps have been provided for that part of unseen ionization distribution. The lowest one is a parabolic and the next one an inverse

parabolic layer step. Their peak densities and the parameter values of the second order polynomial approximations of the other five steps are determined by iteration. The final solution is supposed to result in a least-squares deviation of the extraordinary virtual heights of the profile just derived from the observed extraordinary virtual heights, if ordinary data were used primarily. The number of readings for this selection process is optional. Also, no restrictions have been made as far as the frequency of the first extraordinary reading is concerned. Any formal solution inhibiting a negative gradient is generally rejected.

### 3.2 Valley Problem

#### 3.2.1 No Es-Layer Observed

This problem is being solved similarly to the underlying ionization problem just described. From former studies we know that the semi-thickness of a parabolic approximation of the topside E-layer is about 20 percent thicker than the bottomside semi-thickness. Therefore, we start with such a value for the topside E-layer and apply the underlying ionization procedure after having rejected its lowest (parabolic) step. The lowest plasma density of the valley $f_{min}$ is considered a free parameter between 0.0 and $f_{coE'}$, the ordinary critical frequency of the E-layer.

#### 3.2.2 Es-Layer Observed

##### 3.2.2.1 Es-Layer Within the Topside E-Layer

This problem differs from the above one only slightly. The semi-thickness of the topside E-layer is no longer an unknown parameter and $f_{min}$ needs only to be varied between 0.0 and the plasma frequency at the height of the emerging Es-layer.

##### 3.2.2.2 Es-Layer Within the Valley

In such a case we proceed in two steps. The first one is the determination of the real height of this Es-layer; the second one is similar to the underlying ionization procedure described under 3.2.2.1.

The analysis starts with an unknown semi-thickness of the topside E-layer. It assumes an inverse parabolic layer shape of the electron density distribution within the valley from $f_{min}$ up to the Es-layer level. By iteration, that solution is selected which results in practically equal real heights for the observed Es-layer.

The second part of the analysis allows for a slight discontinuity of the inverse parabolic electron density distribution at the Es-layer level and proceeds in the same way as under 3.2.2.1.

## 4 Conclusions

The present preliminary test results are very promising. In spite of that fact, the author does not want to publish them beforehand; he just wants to mention that the above analysis needs a big computer and a remarkable amount of computer time.

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# APPENDIX D

## Historical Summary

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Two problems aroused my interest in electron density profile calculations, the ambiguity as described by Manning (1949) and the sensitivity of the virtual height to details at the reflection level caused by the singularity of the group refractive index at this point. The latter led immediately to the conclusion that a numerical solution based on an approximation of the profile by a number of linear segments will be slowly convergent (Budden, 1955). This situation was significantly improved by introducing a quadratic variation within each interval with continuous slopes at the interval limits (Paul, 1960a). A more detailed justification for a second (or higher) order approximation together with a study of the effects of the singularity of the group refractive index on the virtual heights was published later (Paul, 1967).

Some theoretical considerations indicated a solution to the ambiguity problem by the joint use of ordinary and extraordinary components (Paul, 1960b). A very general numerical investigation about the relation between ordinary and extraordinary group refractive index showed that in practice the chances for removing the ambiguity depend strongly on latitude (dip angle) and are very low in the vicinity of a dip angle of 39° (Paul and Smith, 1968).

In more recent development a simplified version of profile calculation was derived based on the same principles mentioned above (Paul, 1977). The number of computation steps has been drastically reduced without significant loss of accuracy. This program will be available on the computer of the new HF sounding system for on-line processing.

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# APPENDIX E

## Historical Summary

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Electron density profile inversion work at the Boulder Laboratories began on a relatively large scale shortly after orbiting of the first earth satellite in 1957. We selected Budden's matrix method for this purpose, obtaining copies of the relevant programs from Pennsylvania State University. Our goal was to establish a means by which profiles could be computed for a selection of ionosonde stations in the U.S. and cooperating networks, with the onerous part of the data scaling performed at the stations. By mid 1959, hourly profiles were being computed for current data from eight U.S. stations and a total of 11 were active in 1960. (Wright, 1962) In 1960, the work of Paul came to my attention; three advantages were expected from a reprogramming of our inversion methods according to these ideas: a better approximation to the profile shape with less input data, the prospect of improved corrections for underlying ionization and valley errors, and more accurate treatment of the refractive index at high magnetic dip angles. A new system was in usable form by 1963 (Paul and Wright, 1963, 1964). It is described rather completely in the October 1967 special issue of Radio Science (Paul, 1967; Howe and McKinnis, 1967; Wright, 1967). Some efforts have been made to compare the accuracy of the inversion, and particularly the corrections for underlying D-region ionization and valley effects, by comparison with rocket profiles (Wright and Paul, 1974; Wright et al., 1975). Computation of individual hourly profiles was abandoned in 1962 following the development of a "composite" method of obtaining monthly median profiles (Wright, 1960; Laird and Wright, 1967; Laird, Wright and Gautier, 1974).

More recently, attention has turned to the development of methods by which profiles could be obtained automatically within a digital ionosonde. (Wright et al., 1972).

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APPENDIX F

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L.F.MCNAMARA JUNE 1978


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